Constable's Educational Series.

FIRST STEPS IN ARITHMETIC.

BY JAMES CURRIE, A.M.,

Author of "A Practical Arithmetic for Elementary Schools," "Early and Infant School-Education," etc.

THOMAS LAURIE, EDINBURGH.

Price Sixpence.

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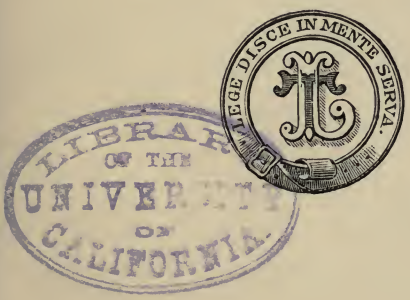
PRINCIPAL OF THE CHURCH OF SCOTLAND TRAINING-COLLEGE, EDINBURGH;
AUTHOR OF "EARLY AND INFANT SCHOOL-EDUCATION,"
"COMMON SCHOOL EDUCATION," ETC.



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PREFACE.

THIS treatise of Arithmetic is designed to comprise all that is needed by the pupils of common schools, and by those of higher schools till they have completed their elementary education.

It is not one of theory, since the instruction of pupils of their standing must be, in the main, practical; nor, on the other hand, is it a mere collection of examples, since the only practical instruction worthy of the name is that which sets the processes before them in a rational way. It aims throughout at that just combination of theory with practice which is the greatest merit of an elementary text-book. The explanations are given concisely, and in the form in which they are likely to be soonest apprehended by the pupil; whilst the exercises for practice will be found to be very numerous and carefully graduated.

In particular, Notation and the four elementary operations, on a satisfactory knowledge of which the pupil's subsequent progress depends, are treated with great fulness. An introductory text-book of Arithmetic should not be a mere condensation of a higher one; it should devote the space which it gains from the omission of certain of the more advanced rules to the ampler treatment of those which are fundamental. Where the arithmetic of a school is weak at all, it is in these rules that the weakness almost invariably lies; and it is in these rules, according to the testimony of all competent authorities, that the most material improvement in the teaching of the subject is to be looked for.

In the arrangement of the treatise the author has kept in view the requirements of the Privy-Council for Elementary Schools and Pupil-Teachers, although he has not limited himself by them.

The Miscellaneous Exercises at the end have been taken chiefly from the papers of the Privy-Council and Dick Bequest Examinations.

For the convenience of junior classes the early chapters, treating of the elementary operations with simple numbers and with money, and forming pp. 1-64 of the present work, are published separately under the title of "First Steps in Arithmetic."

CONTENTS.

TABLES OF MONEY, WEIGHT, AND MEASURES.

MONEY.

I. Money of Account.

4 farthings, *f.*	=	1 penny, *d.*
12 pence	=	1 shilling, *s.*
20 shillings	=	1 pound, £

II. Coins in Circulation.

BRONZE.

2 farthings	=	1 halfpenny, ½*d.*
2 halfpence	=	1 penny.

SILVER.

4 threepenny pieces	=	1 shilling.
3 groats	=	1 shilling.
2 sixpences	=	1 shilling.
2 shillings	=	1 florin, *fl.*
2 shillings and sixp.	=	1 half-crown.
5 shillings	=	1 crown, *cr.*

GOLD.

10 shillings 4 half-crowns 5 florins 2 crowns	=	1 half-sovereign.
20 shillings 3 half-crowns 10 florins 4 crowns	=	1 sovereign.

Paper money is also in use. One pound-note represents the value of 20s., or one sovereign; and there are also five-pound notes, ten-pound notes, twenty-pound notes, fifty-pound notes, and one-hundred-pound notes.

The guinea, formerly a gold coin = £1, 1s., is still recognised as a standard value, though the coin itself is not in use: so the half-guinea, or 10s. 6d.

WEIGHT.

III. Avoirdupois Weight

is used for all common goods.

16 drams, *dr.*	= 1 ounce,	*oz.*
16 oz.	= 1 pound,	*lb.*
28 lb	= 1 quarter,	*qr.*
4 qrs. or 112 lb	= 1 hundredwt.	*cwt.*
20 cwt.	= 1 ton,	*T.*
	Also,	
14 lb	= 1 stone,	*st.*

IV. Troy Weight

is used for weighing the precious metals and jewellery.

24 grains, *gr.*	= 1 pennyweight,	*dwt.*
20 dwt.	= 1 ounce,	*oz.*
12 oz.	= 1 pound,	lb

Note.—The lb Troy = 5760 gr.
The lb Avoir. = 7000 gr.

LENGTH.

V. Lineal Measure

is used for measuring length, and is hence often called *long measure.*

12 inches, *in.*	= 1 foot,	*ft.*
3 feet	= 1 yard,	*yd.*
5½ yards	= 1 pole,	*po.*
40 poles	= 1 furlong,	*fur.*
8 furlongs	= 1 mile,	*ml.*

Tradesmen use what is called a *foot-rule* of three feet long for measuring with, on which the feet are divided into inches, and the inches into eighth parts, tenths, or sixteenths. For longer measurements, a tape or line of 22 yards, similarly divided, is commonly used.

Obsolete measures, but still used for special purposes, are the following:—

1 line	= ⅓th inch.
1 palm	= 3 inches.
1 span	= 9 inches.
1 cubit	= 18 inches.
1 hand (for measuring height of horses)	= 4 inches.
1 fathom (for measuring depth)	= 6 feet.
1 geographical mile	= 1 mile 266 yds. [nearly.
1 league	= 3 geog. miles.
1 degree	= 60 geog. miles.

VI. Cloth Measure

is used for measuring cloth.

2¼ inches	= 1 nail,	*nl.*
4 nails	= quarter,	*qr.*
4 quarters	= 1 yard,	*yd.*
	Also,	
5 quarters	= 1 ell,	*E.*

The draper's rod, one yard long, is divided according to this measure; though in practice, fractions (sixteenths) of a yard are more commonly used.

VII. Land Measure

is used for measuring land. Surveyors use a chain for this purpose, called Gunter's chain, 22 yards (or 4 poles) long, and divided into 100 parts or *links.*

22 yards	= 1 chain of 100 lks.
10 chains	= 1 furlong.

Note.—The link = $7\frac{92}{100}$ inches.

SURFACE.

VIII. Square Measure,

sometimes called *superficial measure*, is used for measuring *surface* or *area*.

144 *sq. in.* = 1 *sq. ft.*
9 sq. ft. = 1 *sq. yd.*
30¼ sq. yd. = 1 *sq. po.* (or perch, *per.*)
40 sq. po. = 1 rood, *ro.*
4 roods = 1 acre, *ac.*
640 acres = 1 *sq. mi.*

Still used for special purposes are the following measures :—

100 sq. feet = 1 square of flooring.
272½ sq. ft. or 1 sq. po. = 1 rod of brickwork.
36 sq. yd. = 1 rood of building.

Land-surveyors, as stated above, use the chain of 100 links, though they express the result of their measurements in this table :—

10,000 square links = 1 square chain.
10 square chains = 1 acre.

SOLIDITY.

IX. Cubic Measure

is used for measuring the contents of *solid bodies, e.g.*, masses of stones or earth (hence often called *solid measure*), or of bodies which have the *shape* of solids, *e.g.*, rooms, cisterns, etc.

1728 cubic in. = 1 cubic ft.
27 cubic ft. = 1 cubic yd.

Shipping is measured by tonnage, timber by loads, and general goods sometimes by barrel-bulk, thus :—

42 cub. ft. = 1 ton shipping, *T. sh.*
40 cub. ft. rough timber, 50 do. hewn = 1 load, *lo.*
5 cub. ft. = 1 barrel-bulk, *B.B.*

CAPACITY.

X. Measure of Capacity

is used for the measurement of liquids, and also of dry goods, like grain, etc.

4 gills, *gi.*	= 1 pint,	*pt.*
2 pints	= 1 quart,	*qt.*
4 quarts	= 1 gallon,	*gal.*
2 gallons	= 1 peck,	*pk.*
4 pecks	= 1 bushel,	*bu.*
8 bushels	= 1 quarter,	*qr.*

The peck, bushel, and quarter are used for dry goods only.

For wine and beer, casks of various sizes are used, of which the most common are—

FOR WINE.

The puncheon	= 84 gal.
The pipe	= 126 gal.
The tun	= 252 gal.

FOR BEER.

The kilderkin	= 18 gal.
The barrel	= 36 gal.
The hogshead, *hhd.*	= 54 gal.

But these casks are not standard measures, and vary in their capacity.

The imperial gallon contains 277·274 cubic inches.

TIME.

XI. Measure of Time.

60 seconds, *sec.*	= 1 minute,	*min.*
60 minutes	= 1 hour,	*ho.*
24 hours	= 1 day,	*da.*
7 days	= 1 week,	*wk.*
52 wks. 1 day, or 365 days	= year,	*yr.*
366 days	= 1 leap year.	
100 years	= 1 century.	

The year is divided into 12 *calendar* months :—

January	31 days	July	31 days
February	28	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

Every year (with very rare exceptions) whose number is divisible by 4, is a leap year; in which February has 29 days.

Thirty days have September,
April, June, and November:
All the rest have thirty-one,
Excepting February alone,
Which has but twenty-eight days clear,
And twenty-nine in each leap year.

The lunar month = 29 da. 12 ho. 44 min.
The solar year = 365 da. 5 ho. 48 min. 48 sec., *i.e.*, nearly 365 days 6 hours (the Julian year).

QUARTERLY TERMS.

In England.

Lady-Day,	March	25.
Midsummer,	June	24.
Michaelmas,	Sept.	29.
Christmas,	Dec.	25.

In Scotland.

Candlemas,	Feb.	2.
Whitsunday,	May	15.
Lammas,	Aug.	1.
Martinmas,	Nov.	11.

The centuries are reckoned, among Christian nations, in numerical order from the birth of our Lord (called the Christian era): thus the years 1 to 99 are the first century, 100 to 199 the second, and so on. This is the nineteenth century. Any particular year, *e.g.*, 1864, is denoted 1864 A.D., *i.e.*, *Anno Domini, in the year of our Lord.* The years before the birth of our Lord are reckoned back in order from that event: thus 1460 A.C., means *Ante Christum*, or *before Christ.*

INCLINATION.

XII. Angular Measure

is used for measuring the *angle* or inclination of one line to another.

60 seconds, ″	= 1 minute,	′
60′	= 1 degree,	°
90°	= 1 right angle,	L
360°	= 1 circle,	⊙

The following Tables are subjoined for reference:—

Paper Measure.

24 sheets	= 1 quire,	*qu.*
20 quires	= 1 ream,	*re.*
21½ quires	= 1 perfect ream.	

Cloth Measure.

5 quarters	= 1 English ell.
3 quarters	= 1 Flemish ell, *Fl. E.*
6 quarters	= 1 French ell, *Fr. E.*
37 inches	= 1 Scotch ell, *S. E.*

Apothecaries' Weight.

OLD MEASURE.

20 grains, *gr.*	= 1 scruple,	℈
3 scruples	= 1 drachm,	ʒ
8 drachms	= 1 ounce Troy,	℥
12 ounces	= 1 ℔ Troy.	

NEW MEASURE (1862).

437½ grains = 1 ounce Avoir.

Apothecaries' Fluid Measure.

60 minims, ♏	= 1 fluid drachm,	*f.* ʒ
8 fl. drachms	= 1 fluid ounce,	*f.* ℥
16 ounces	= 1 ℔	
20 ounces	= 1 pint,	O
8 pints	= 1 gallon,	C

FOREIGN MONEY.

United States.

10 cents	= 1 dime.
10 dimes	= 1 dollar, $

1 dollar = 4s. 2d.

France.

100 centimes = 1 franc.
1 franc = 9½d. nearly.

Canada.

Accounts are kept in £ s. d. *currency*, of which £1 = 16s. 8d. sterling.

East Indies.

16 annas = 1 rupee.
1 rupee = 1s. 10½d.

OLD SCOTCH MONEY AND MEASURES

still recognised in Scotland for certain purposes.

Money.

1 shilling Scots	= 1d. sterling.
£1 Scots	= 1s. 8d. do.

being one-twelfth of the same names sterling.

1 merk = 1s. 1⅓d.

Lineal Measure.

37 inches	= 1 ell.
6 ells	= 1 fall.
4 falls	= 1 chain.

1 chain = 1⅓ Imp. chain nearly.

Square Measure.

36 sq. ells	= 1 square fall.
40 sq. falls	= 1 rood.
4 roods	= 1 acre.

1 acre = 1¼ Imp. acre nearly.

Liquid Measure.

4 gills	= 1 mutchkin.
2 mutchkins	= 1 chopin.
2 chopins	= 1 pint.
8 pints	= 1 gallon.

1 gallon = 3 Imp. gallons nearly.

Dry Measure.

4 pecks	= 1 firlot.
4 firlots	= 1 boll.
10 bolls	= 1 chalder.

The *Wheat Firlot* was nearly equal to an Imp. bushel (= ·998 bush.); the *Barley Firlot* nearly equal to 1¼ bush. (= 1·456 bush.) The *Boll* weighs 140 ℔ Avoir.

PROPOSED DECIMAL COINAGE.

1 mil = one thousandth part of £1,
or = ¼d. less $\frac{1}{24}$d.

10 mils = 1 cent, one-hundredth of £1.
10 cents = 1 florin, one-tenth of £1.
10 florins = £1.

NUMERATION AND NOTATION.

1. Numbers of One Place.

One finger and *one* finger make *two* fingers.
Two fingers and *one* finger make *three* fingers.
Three fingers and *one* finger make *four* fingers.
Four fingers and *one* finger make *five* fingers.
Five fingers and *one* finger make *six* fingers.
Six fingers and *one* finger make *seven* fingers.
Seven fingers and *one* finger make *eight* fingers.
Eight fingers and *one* finger make *nine* fingers. *Bf.*[1]

One, two, three, four, five, six, seven, eight, nine, are the *names* of *numbers* used in *counting.*

The *naming* of numbers is called *Numeration.*

One, three, five, seven, nine, are called *odd* numbers.

Two, four, six, eight, are called *even* numbers.

These nine numbers mean so many *ones,* or *units* as they are called ; thus *two* means two *ones* or two *units, three* means three *ones* or three *units,* and so on.

EXERCISE I. *Bf.*

1. Repeat the table of units, as given above.
2. Repeat it, using balls, marbles, boys, etc., instead of fingers.
3. Repeat it with the numbers alone, thus, "one and one are two."
4. Count from one up to nine, and from nine back to one.
5. Count the odd numbers from one to nine ; from nine to one.
6. Count the even numbers from two to eight ; from eight to two.
7. Name the two numbers next above five, eight, three, etc.[2]
8. Name the two numbers next below six, nine, four, etc.
9. Hold up three fingers, five, seven, etc.
10. How many wheels has a cart? How many halfpence in a penny? How many pence in a threepenny-piece? How many letters in the word "dog"? How many legs has a cow? etc.
11. If I have four pence and give one away, how many do I keep? If I have six marbles, and get one from James, how many have I? etc.

2.

The nine numbers are denoted by signs or *figures,* thus :—

one,	two,	three,	four,	five,	six,	seven,	eight,	nine,
1	2	3	4	5	6	7	8	9

The *figuring* of numbers is called their *Notation.*

[1] *Bf.* means that the ball-frame may be used for illustration.
[2] *Etc.* means that various other questions of the same kind may be given.

EXERCISE II.

1. Write down the figures—(1.) even along; (2.) up and down.
2. Name the numbers in Ex. iv. sect. 16.
3. Write down the figures for the same numbers.[1]

3. Numbers of Two Places.

If I count nine on my fingers, I find one finger over.

Nine fingers and *one* finger make *ten* fingers; which is the whole number of them.

If I wish to count beyond ten, I must begin again and go round a second time; that will give me two-times ten or *two tens.* Three times round will give three-times ten or *three tens;* and so on, up to nine-times round, which will give nine-times ten or *nine tens.*

One ten	is called	*Ten,*	denoted by	10.
Two tens	are „	*Twenty,*	„	20.
Three tens	„	*Thirty,*	„	30.
Four tens	„	*Forty,*	„	40.
Five tens	„	*Fifty,*	„	50.
Six tens	„	*Sixty,*	„	60.
Seven tens	„	*Seventy,*	„	70.
Eight tens	„	*Eighty,*	„	80.
Nine tens	„	*Ninety,*	„	90.

The *tens* are numbers *of two places.* They are denoted by the figures for the units with a *cipher* on the right.

The value of a figure is increased *ten times* by its being written in the *second place* from the right: thus 3 denotes three units, but 30 denotes three tens. Hence the notation we use is called the *decimal*[2] notation.

The cipher is used to fill up the *first* or right-hand place, when that place contains no units or nothing; hence it is commonly called *nought* or *nothing.* It is never used alone.

EXERCISE III.

1. Repeat the table of tens; backwards; by odds; by evens.
2. Count the tens.
3. Name the tens next above forty, sixty, etc.; next below thirty, eighty, etc.
4. How many fingers have six boys? eight boys? etc. *Bf.*
5. How many boys together have thirty fingers? seventy? etc. *Bf.*
6. How many units in eight tens? six tens? etc.
7. How many tens in thirty units? in seventy units? etc.

[1] Either from the copy or to dictation. The teacher may vary the exercise by having the figures pointed out on the board from columns written by himself.

[2] From the Latin word *decem,* ten.

8. If I have ninety marbles and give away ten, how many do I keep? If I have seventy, and get ten more, and other ten, how many have I? etc.

9. Write down the figures for the tens below each other.

10. Name the numbers, Ex. vi. sect. 17, Nos. 1, 2.

11. Write down the figures for these numbers.

4.

One ten and one unit are called	*eleven*,	denoted by		11
One ten and two units	„ *twelve*,	„	.	12
One ten and three units	„ *thirteen*,	„	.	13
One ten and four units	„ *fourteen*,	„	.	14
One ten and five units	„ *fifteen*,	„	.	15
One ten and six units	„ *sixteen*,	„	.	16
One ten and seven units	„ *seventeen*,	„	.	17
One ten and eight units	„ *eighteen*,	„	.	18
One ten and nine units	„ *nineteen*,	„	.	19

The *tens-units* are also numbers *of two places;* the first being the units' place, the second the tens' place.

The names of the numbers from 13 to 19 are formed by putting the number of the units before that of the tens; thus *thirteen* is *three and ten*, *fourteen* is *four and ten*, etc. The names of all the other numbers of two places are formed by putting the number of the tens before that of the units; thus

Two tens and one are called	*twenty-one*,	denoted by		21
Two tens and two	„ *twenty-two*,	„	.	22
Etc.	etc.			etc.
Three tens and one	„ *thirty-one*,	„	.	31
Three tens and two	„ *thirty-two*,	„	.	32
Etc.	etc.			etc.

When numbers of two places are written below each other, units are written below units, and tens below tens.

EXERCISE IV.

1. Repeat the table of tens-units fron ten to twenty, from twenty to thirty, etc.

2. Count the tens-units from ten to twenty, from twenty to thirty, etc.

3. If one boy holds up the fingers of his right hand, and other three boys all their fingers, how many fingers are up? how many if another boy holds up his? if another? if one boy removes his? etc., *Bf.*

4. If I hold up seven fingers, how many girls must hold up all their fingers to make twenty-seven? to make thirty-seven? etc., *Bf.*

5. Count by tens from thirty-one, from forty-two, etc.
Count by tens back from ninety-eight, eighty-seven, etc.

6. How many are 1 ten and 4? 2 tens and 6? 4 tens and 7? etc.

7. What tens and units make up 18, 27, 33, 47? etc.

8. Figure from ten to twenty, twenty to thirty, etc.

9. Figure 2 tens below 2 units, 3 tens below 3 units, etc. ; 9 units below 9 tens, 8 units below 8 tens, etc,
10. Name the numbers in Ex. vi. sect. 17, No. 3-25.
11. Write down, or tell in order, the figures for these numbers.

5. Numbers of Three Places.

Nine tens and *one ten* make *ten tens.*

As we put ten units together, and call them one-ten, so we put the ten-tens together and call them *one hundred. Bf.*

One hundred is denoted by . 100
Two hundreds „ . . 200
Three hundreds „ . . 300, and so on.

The *hundreds* are numbers *of three places.* They are denoted by the figures for the units with *two ciphers* on the right.

The value of a figure is increased a *hundred times* by its being written in the *third place ;* thus 3 denotes three units, but 300 denotes three hundreds.

The two ciphers are used to fill up the first and second places, when these places contain no units and no tens.

EXERCISE V.

1. Count the hundreds, backwards, by odds, by evens.
2. Name the numbers in Ex. ix. sect. 19, Nos. 1, 2.
3. Tell in order the figures in these numbers.
4. How many tens in 100, 500, 800? etc.
5. How many hundreds in 10 tens, 70 tens? etc.
6. Figure the hundreds in an up-and-down line.
7. Figure 1 hund. below 1 ten, 2 hund. below 2 tens, etc.
   9 tens below 9 hund., 8 tens below 8 hund., etc.
8. Figure 1 h. below 1 t. below 1 u.—2 h. below 2 t. below 2 u. etc.
   9 u. below 9 t. below 9 h.—8 u. below 8 t. below 8 h. etc.
9. Write down the figures for the numbers Quest. 2.

6.

Numbers consisting of hundreds, tens, and units are also numbers *of three places ;* the first being the units' place, the second the tens' place, and the third the hundreds' place.

Their names are formed by combining, in their order, the number of the hundreds, the number of the tens, and the number of the units. Thus—

146 denotes 1 h. 4 t. 6 u., and is called one hundred and forty-six.
270 „ 2 h. 7 t. 0 u., „ two hundred and seventy.
804 „ 8 h. 0 t. 4 u., „ eight hundred and four.

Where there are no units, or no tens, these are omitted in the names, as in the last two numbers.

When numbers are written in column, the same places must be kept below each other.

EXERCISE VI.

1. Count from one hundred to nine hundred and ninety by tens, and from nine hundred and ninety to one hundred by tens.
2. Count from two hundred and forty to two hundred and fifty.
,, five hundred and sixty to five hundred and seventy, etc.
3. Name the numbers in Ex. ix. sect. 19, No. 3-25.
4. Tell in their order the figures in these numbers.
5. Figure below each other two hundred and twenty-two, two hundred and two, two hundred and twenty, two hundred, twenty, two :—etc. Repeat the same, beginning with the units.
6. Figure the numbers in Quest. 3.

7. Numbers of One Period.

All numbers of one, two, or three places—that is, all numbers from 1 to 999—are numbers *of one period.*

Numbers of one place may be written with their period completed by putting two ciphers to the left hand. Thus, since 6 units is the same as 0 hundreds 0 tens 6 units, the number 6 may be written 006, and read no hundred and six.

Numbers of two places may be written with their period completed by putting one cipher to the left hand. Thus, since 6 tens 5 units is the same as 0 hundreds 6 tens 5 units, the number 65 may be written 065, and read no hundred and sixty-five.

A cipher placed to the *left* hand of any figure does not alter its *place*, nor, consequently, its *value*.

EXERCISE VII.

1. What are the numbers whose figures in order are three, two, one ; four, nothing, six ; six, four ; seven, two, nothing, ? etc.
2. What figures in order denote two hundred, two hundred and six, five hundred and thirty-two ? etc.
3. What are these numbers made up of ?—Ex. ix. sect. 19.
4. Figure their several parts in order below each other ?
5. Point out the tens' place in them ? units' place ? hundreds' ?
6. What numbers are made up of these parts, 3 h. 2 t. 6 u. ? 4 h. 0 t. 7 u. ? 7 h. 4 t. 0 u. ? 8 h. 4 u. ? etc.
7. Read these numbers, 7, 17, 20, 34, etc. (1.) as they stand ; (2.) with their periods filled up ?
8. Read these numbers, 008—080—800—088—880—80, etc.
9. Take any number, as 5. What does it denote with one nought before it ? with two ? with one after it ? with two ? with one before and one after it ? Which nought increases its value ten times ? which leaves it unaltered ? What two noughts increase its value one hundred times ? what two leave it unaltered ? What two increase its value ten times ? etc.

10. Write the numbers, eight, ten, twenty-five, etc. (1.) as incomplete periods; (2.) as completed periods.[1]

11. Write in figures: fifty-three, thirty-seven, ninety-four, one hundred and seventy, four hundred and sixty-nine, eight hundred and eight, seven hundred and fourteen, seventy-eight, two hundred and eighteen, five hundred and five, six hundred and sixty, three hundred and thirty-three, nine hundred and forty one, five hundred and sixteen, etc.

*** *When the pupil has obtained perfect facility in reading and writing numbers of one period, he may proceed with their addition, subtraction, and multiplication, returning afterwards to the notation of larger numbers.*

8. Numbers of Two Periods.

Nine hundreds and *one hundred* make ten hundreds. As we put ten tens together and call them one hundred, so we put the ten hundreds together and call them *one thousand.*

One thousand is denoted by . . .	1,000
Two thousands, ...	2,000
Ten thousands, ...	10,000
Eleven thousands, ...	11,000
One hundred thousand, . . .	100,000
Three hundred and forty-seven thousand, .	247,000

Any number of thousands is written as if it were units, with three ciphers on the right.

If the number contain also hundreds–tens–units, these are written in place of the cyphers. Thus—

One thousand five hundred is denoted by .	1,500
Two thousand six hundred and thirty, .	2,630
Ten thousand four hundred and twenty-five,	10,425
Eleven thousand seven hundred and eight, .	11,708
One hundred thousand one hundred and thirty,	100,130
Three and forty-seven thousand three hundred and forty seven, ...	347,347

Every number of thousands has from four to six places, forming *two periods.* The *first* period containing the hundreds–tens–units, if there are any; the *second* the thousands.

*** The two periods are often separated by a comma, as above, to prevent mistakes in reading numbers; but by practice the pupil will soon be able to do without it.

EXERCISE VIII.

1. Read the numbers, Ex. x. sect. 20.
2. Write to dictation the numbers in same Exercise.
3. In 501274 (or any of the numbers in same Exercise), how many thousands? hundreds? tens of thousands? units? hundred thousands? tens?

[1] Counters may be used to aid the pupil in writing numbers of one period; see Note, section 9.

4. In 347029 (or any of the numbers in same Exercise), what does the 3 denote? the 9? 0? 7? 4? 2?

5. What figures in order denote six thousand three hundred? or any of the numbers in the same Exercise?

6. What numbers are denoted by the following sets of figures in order, 4, 2, 4, 8? 8, 0, 7, 9, 2? 3, 6, 5, 2, 0, 1? etc.

9. Numbers of Three Periods.

Nine hundred thousands and *one hundred thousands* make a *thousand thousands*, which we call one *Million*.

One million is denoted by . . .	1,000,000
Two millions are	2,000,000
Ten millions, ...	10,000,000
Eleven millions, ...	11,000,000
One hundred millions, ...	100,000,000
Three hundred and forty-seven millions, .	347,000,000

Any number of millions is written as if it were units, with six ciphers to the right.

If the number contain also thousands, hundreds, tens, and units, these are written in place of the ciphers, thus:—

One million five hundred thousand is denoted by	1,500,000
Two millions six hundred and thirty thousand,	2,630,000
Ten millions four hundred and twenty-five thousand,	10,000,000
Eleven millions seven hundred and eight thousand five hundred and ten, ...	11,708,510
One hundred millions one hundred thousand and one hundred, ...	100,100,100
Three hund. and forty-seven mills. three hund. and forty-seven thousand, three hun. and forty-seven,	347,347,347

Every number of millions has from seven to nine places, forming *three periods;* the first called the units' period, the second the thousands', and the third the millions'.

EXERCISE IX.

1. Read the numbers, Ex. xii. sect. 21.

2. In 243,076,549 (or any of the above numbers), how many hundreds? tens of thousands? tens of millions? units? hundreds of thousands? etc.

3. In 804395276 (or any of the above numbers), what does the 5 denote? 4? 8? 0? 6? 7? etc.

4. What figures in order denote seven millions and thirty thousand, or any of the above numbers?

5. What is denoted by the 1st place, 2d period? 2d place, 1st period? 1st place, 3d period? 2d period? 1st period? 3d place, 1st period? etc.[1]

6. Write to dictation the numbers, Ex. xii. sect. 21.

[1] This questioning may be continued with the help of three periods of counters; thus ●●● ●●● ●●●

These may be also advantageously used in the following exercises in dicta-

10. Numbers of more than Three Periods.

Nine hundred millions and one hundred millions make a thousand millions.

One thousand millions are denoted by 1,000,000,000
Ten thousand millions, . . . 10,000,000,000
A hundred thousand millions, . . 100,000,000,000

Thousands of millions are written as if they were thousands, and six ciphers are added.

If there are also millions, thousands, and units, these are written in place of the ciphers, thus :—

One thousand two hundred and thirty millions is 1,230,000,000
Ten thousand five hundred and sixteen millions,
five hundred and sixteen thousand, . 10,516,516,000
One hund. and thirty-seven thous., one hund. and
thirty-seven mills., one hund. and thirty-seven
thousand one hundred and thirty-seven, 137,137,137,137

Every number of thousands of millions contains from ten to twelve places, forming four periods; which may be separated by commas, as above.

Still larger numbers may be expressed by a fifth period, commencing at a million of millions, or, as it is called, a *Billion;* or even a sixth period for thousands of billions, thus :—

B. M. U.

137,137,137,137,137,137

But numbers of more than three periods rarely occur.

11. Appendix on the Roman Notation.

Numbers are sometimes denoted by another set of characters, called *Roman.*[1]

These are seven in number, thus :—

1 is denoted by the letter I, 5 by V, 10 by X, 50 by L, 100 by C, 500 by D, and 1000 by M.

EXERCISE X.

1. Name the letters, with the numbers they denote.
2. Write down the letters, with the numbers they denote.

tion. Thus the pupil may be asked to read 28 14 7, or to write numbers in that way in the first instance, and then to supply the necessary ciphers.

[1] So called from having been used in the ancient Roman notation. The ordinary characters are often spoken of as the Arabic, from having come to us through the Arabs.

12. To denote other numbers, these seven characters are combined in two ways—*First*, a character *following* another of greater or equal value *adds* thereto its own value; thus VI denotes 5+1, or 6. *Second*, a character *preceding* another of greater value subtracts therefrom its own value; thus IV denotes 5 − 1, or 4.

The only numbers which are denoted by subtraction are the units next under V and X, and the tens next under L and C; thus 4 is denoted by IV, 9 by IX, 40 by XL, and 90 by XC. All the rest are denoted by addition.

I	1	X	10	XI	11	C	100	CX	110	M	1000
II	2	XX	20	XII	12	CC	200	CXX	120	MC	1100
III	3	XXX	30	XIII	13	CCC	300	CXXIV	124	MCC	1200
IV	4	XL	40	XIV	14	CCCC	400	CXLIX	149	MD	1500
V	5	L	50	XV	15	D	500	CCXXX	230	MDLXIV	1564
VI	6	LX	60	XLI	41	DC	600	CCCLXI	361	MDCX	1610
VII	7	LXX	70	XLII	42	DCC	700	DXC	590	MDCXCII	1692
VIII	8	LXXX	80	XLIII	43	DCCC	800	DCCIII	703	MDCCC	1800
IX	9	XC	90	etc.		DCCCC	900	etc.		MM	2000

The Roman characters are now used *only to denote numbers, e.g.*, the chapters of a book, the hours on the clock, the houses in a street, and the years; never to calculate with.

EXERCISE XI.

1. What numbers are denoted by V, X, IV, XX, XXII, XL, etc.?
2. Name, or write down, letters for the numbers, Ex. iv. sect. 16.
3. Name, or write down, letters for the numbers, Ex. vi. sect. 17.
4. Name, or write down, letters for the numbers, Ex. ix. sect. 19.
5. Do. do. 1250, 1365, 1473, 1582, 1624, 1738, 1806, 1835, 1864.

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13. ADDITION.

Ex.—Of four flocks of sheep, one contained 35, the second 29, the third 50, and the fourth 47. They were put into one field; how many sheep were there in all?

Here we have to find one number as large as four given numbers together.

The number to be found is called the *sum*.

The sum is got by *adding* the four given numbers together.

The process of adding is called *addition;* and—when the things to be added are of one kind, as here—*simple addition.*

The sign of addition is + (plus): thus 1 + 1 are 2.

We cannot find the sum of the above four numbers at once; they are too large. We must therefore add them *in parts;* for which purpose we must learn the addition of the first nine numbers.

14. Addition Table.

*** This Table should be learnt first in lines *even along*; thus, 1 and 1 are 2; 2 and 1 are 3, etc.; afterwards in lines *up and down*. *Bf.*

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
2 ... 3	2 ... 4	2 ... 5	2 ... 6	2 ... 7	2 ... 8	2 ... 9	2 ... 10	2 ... 11
3 ... 4	3 ... 5	3 ... 6	3 ... 7	3 ... 8	3 ... 9	3 ... 10	3 ... 11	3 ... 12
4 ... 5	4 ... 6	4 ... 7	4 ... 8	4 ... 9	4 ... 10	4 ... 11	4 ... 12	4 ... 13
5 ... 6	5 ... 7	5 ... 8	5 ... 9	5 ... 10	5 ... 11	5 ... 12	5 ... 13	5 ... 14
6 ... 7	6 ... 8	6 ... 9	6 ... 10	6 ... 11	6 ... 12	6 ... 13	6 ... 14	6 ... 15
7 ... 8	7 ... 9	7 ... 10	7 ... 11	7 ... 12	7 ... 13	7 ... 14	7 ... 15	7 ... 16
8 ... 9	8 ... 10	8 ... 11	8 ... 12	8 ... 13	8 ... 14	8 ... 15	8 ... 16	8 ... 17
9 ... 10	9 ... 11	9 ... 12	9 ... 13	9 ... 14	9 ... 15	9 ... 16	9 ... 17	9 ... 18

EXERCISE I. *Bf.*

1. Repeat the several lines of the table even along; backwards; by odds and evens.
2. Repeat the several lines up and down in the same orders.
3. 5 and 6 are —? 8 and 3 are —? 4 and 9 are —? etc.
4. 2 + 3 + 5 are —? 6 + 3 + 8 are —? etc.[1]
5. 2 + 4 + 3 + 7 are —? 5 + 2 + 2 + 6 are —? etc.[1]
6. 2 books and 3 books are —? I have 5d. and John 4d., how much have we both? John had 3 marbles; if he bought 6 and gained 7, how many has he now? etc.
7. Write down the columns of the table in order.

15.

If one of the numbers to be added contains tens and units, add the units as if they were alone, and prefix the number of tens. Thus—

11 and 1 are 12; 12 and 1 are 13; 13 and 1 are 14.
11 and 2 are 13; 12 and 2 are 14; 13 and 2 are 15.
Etc. etc. etc.

EXERCISE II.

1. Repeat the several lines of this table from 11 to 19, (1.) even along, (2.) up and down.
2. Repeat a similar table for 21-29, 31-39, etc.
3. 11 and 4 are —? 17 and 8 are —? etc.
4. 5 + 19 + 4 are —? 17 + 6 + 5 are —? etc.
5. 16 + 7 + 2 + 4 are —? 13 + 4 + 9 + 6 are —? etc.
6. Write down any line of this Table in order.

EXERCISE III.

Count forward from 1, 2, 3, 4, 5, 6, 7, 8, 9 by *twos*, then by *threes*, *fours*, etc., up to *nines*.

[1] In Ques. 4, the sum of the first two numbers, and in Ques. 5, the sum of the first three, should not exceed nine.

16. Addition of Numbers of One Place.

Ex.—John had 8 marbles, James had 4, William had 7, and Henry 5 ; how many had they amongst them ?

We can find the sum of these small numbers without writing ; but if we wish to write down the process, we set the numbers below each other, and add step by step, thus—

(5 and 7 are) 12 ; (and 4 are) 16 ; (and 8 are) 24—which is the sum required.

8
4
7
5
—
24

*** The words within parentheses may be used for some time by the pupil, but should be omitted at the earliest moment he can do without them.

The addition may be proved to be correct by adding the column downwards from the top. The sum of any series of numbers is the same in whatever order they are added.

EXERCISE IV.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(1.)	8	9	2	6	8	3	5	6	7	2	1	4	5	6	7	8	9	2
(2.)	7	4	5	1	6	2	9	7	2	0	1	3	6	0	5	9	0	4
(3.)	5	7	7	5	4	1	8	7	5	6	9	2	0	5	4	1	3	2
(4.)	6	6	8	4	8	6	4	9	4	3	2	1	0	8	6	9	3	1
(5.)	4	0	9	3	5	3	8	7	2	0	1	5	6	8	5	6	9	8
(6.)	3	8	1	2	4	0	0	8	3	6	9	4	5	9	1	7	3	4
(7.)	2	5	0	7	2	9	5	0	8	6	5	3	1	2	3	4	0	1
(8.)	9	4	6	0	3	7	6	2	0	4	9	3	4	1	5	7	3	0
(9.)	0	1	5	9	5	4	3	2	1	0	1	2	3	4	5	6	7	8
(10.)	5	1	4	4	4	5	6	7	8	9	8	7	6	5	4	3	2	1
(11.)	4	5	3	5	0	1	2	3	4	5	6	7	8	9	0	4	3	8
(12.)	8	9	2	2	6	1	0	5	3	4	7	4	3	8	5	2	2	9
(13.)	7	6	5	1	7	0	5	6	4	3	9	2	1	8	4	6	3	8
(14.)	3	2	7	2	6	4	5	4	0	9	1	2	3	5	8	2	0	9
(15.)	5	0	6	0	4	1	0	7	3	6	5	4	2	9	2	1	1	7
(16.)	8	7	7	9	1	4	7	8	2	1	5	3	9	4	3	6	6	4
(17.)	2	6	8	3	0	6	7	5	4	8	6	2	0	1	4	7	2	5
(18.)	9	5	3	4	8	2	4	3	2	6	0	9	5	4	3	2	0	1
(19.)	5	1	5	6	1	2	4	6	8	0	2	4	6	8	0	2	4	6
(20.)	7	8	4	7	1	3	5	7	9	1	3	5	7	9	1	3	5	7
(21.)	4	2	2	9	4	2	3	6	2	4	7	3	5	8	4	6	9	5
(22.)	0	4	3	0	5	4	3	4	7	8	9	5	6	8	2	0	0	1
(23.)	6	6	1	1	3	4	9	6	8	2	0	1	4	3	7	7	6	4
(24.)	8	9	7	2	5	6	2	1	4	9	3	2	0	8	6	4	3	2
(25.)	1	8	8	3	4	5	8	9	1	4	7	6	8	5	1	2	3	0

*** These numbers may be added in parts of columns, or in whole columns, up—down—from left to right—from right to left. And the pupil should work at them a little every day till he attains expertness in adding.

17. Addition of Numbers of Two Places.

The Table given, sect. 14, serves also for the addition of tens, thus :—

If 1 and 1 are 2, 1 ten and 1 ten are 2 tens, or 10 and 10 are 20.
2 and 1 are 3, 2 tens and 1 ten are 3 tens, or 20 and 10 are 30.
Etc. etc. etc.

EXERCISE V.

Perform Ex. i. Quests. 1-5, with tens.

Ex.—Of four flocks of sheep one contained 35, the second 29, the third 50, and the fourth 47. They were put into one field : how many sheep were there in all ?

Set the numbers below each other in their places. Then in the units' column : (7 and 9 are) 16, (and 5 are) 21 (units ; set down) 1 (in the units' place), and carry 2 (tens to the tens' column). Next, in the tens column : (2 and 4 are) 6, (and 5 are) 11, (and 2 are) 13, (and 13 are) 16 (tens. Set down the) 6 (in the) tens' (column), and (the ten tens as) 1 hundred (in the hundreds' column).

	35
	29
	50
	47
	——
	161

EXERCISE VI.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
20	70	46	23	14	22	34	54	72	29	13	27	41	64	39
30	40	50	13	43	19	96	34	49	64	70	91	27	36	44
40	7	64	14	50	47	94	18	81	49	17	9	4	51	63
50	20	36	29	69	90	25	60	70	80	90	40	56	4	20
60	60	45	56	24	47	18	26	43	31	83	7	91	54	7
40	9	69	73	38	58	37	48	62	15	24	19	9	48	17
70	80	92	20	41	64	74	51	64	82	39	24	47	64	8
80	10	87	34	76	92	82	27	39	51	63	75	87	99	9
90	5	71	47	92	10	45	14	17	20	23	6	9	2	49
30	50	25	56	85	86	37	35	38	41	44	47	50	53	80
40	30	34	81	24	48	29	94	91	87	84	62	59	72	27
50	40	28	73	37	35	15	62	59	18	60	53	27	9	93
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

16. 76+18+37+9+11+24+32+47+3+16+28+76+49+60.
17. 22+80+6+12+15+93+27+36+48+51+70+10+29+8.
18. 37+45+15+7+1+27+39+82+99+4+54+37+10+29.
19. 28+57+3+30+17+37+90+25+41+8+59+32+87+40.
20. 29+5+16+34+64+72+19+7+38+64+28+11+58+38.
21. 18+90+21+7+9+8+15+27+47+50+62+71+89+69.
22. 30+54+4+23+93+47+50+41+39+8+17+28+60.
23. 16+84+17+30+85+74+32+91+11+22+50+5+15+66.
24. 93+9+8+17+29+40+57+85+36+21+73+17+76+82.
25. 87+53+20+6+9+14+65+89+53+28+70+38+67+2.

EXERCISE VII.

1. 10+11 are —? 10+12 are —? 10+13 are —? 10+21 are —? etc.
2. 20+11 are —? 20+12 are —? 20+13 are —? 20+21 are —? etc.
3. 30+11 are —? 30+12 are —? 30+13 are —? 30+21 are —? etc.
4. 40+11 are —? 40+12 are —? 40+13 are —? 40+21 are —? etc.
5. Add the remaining tens in a similar way.
6. 50+25 are —? 20+18 are —? 40+29 are —? etc.
7. 22+15 are —? 34+18 are —? 75+24 are —? etc.

*** In this last question, it is easier to add the tens first; thus: 34+18 are 4 tens and 12, that is 52.

18. Addition of Numbers of One or more Periods.

The table given, section 14, serves also for the addition of hundreds, thousands, etc.; thus,

If 1 and 1 are 2, 1 h. and 1 h. are 2 hs., or 100 and 100 are 200.
2 and 1 are 3, 2 h. and 1 h. are 3 hs., or 200 and 100 are 300.
Etc. etc. etc.

EXERCISE VIII.

Perform Ex. i. Questions 1-5, with hundreds.

Ex.—Four heaps of bricks were lying in a field. The first contained 208 bricks, the second 349, the third 160, and the fourth 87; how many bricks were there in all?

Set the numbers below each other in their places.
In the units' column—(7 and 9 are) 16, (and 8 are) 23 (units; set down) 3 (in the units' place), and carry 2 (tens).
In the tens' column—(2 and 8 are) 10, (and 6 are) 16, (and 4 are) 20; (set down) 0 (in the tens' place) and carry 2 (hundreds).
In the hundreds' column—(2 and 1 are) 3, (and 3 are) 6, (and 2 are) 8, (set down 8 in the hundreds' place).
Sum, 804.

208
349
160
87
804

*** After some practice in adding, the words within parentheses should be omitted.

Rule.—Set the numbers below each other in their places; and add the columns in their order from the units, carrying the tens.

19. EXERCISE IX.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
100	200	418	524	638	793	814	701	649	547	890	736
300	500	296	615	800	215	427	593	524	64	47	624
500	900	306	500	524	300	324	414	700	147	562	93
700	100	851	924	357	618	650	710	810	291	50	14
900	300	628	705	184	509	379	327	81	17	900	257
400	800	435	396	225	493	800	967	47	364	73	39
600	600	200	527	604	215	930	413	913	84	654	572
800	700	753	713	593	336	247	258	27	913	209	809

13. 365+210+93+27+110+345+563+207+824+85+127.
14. 241+56+37+256+357+842+506+37+81+190+429.
15. 306+194+516+70+7+829+593+601+72+720+18.
16. 501+600+60+372+144+11+111+29+360+306+71.
17. 76+706+760+370+307+37+377+84+804+840+9.
18. 275+360+910+989+724+57+507+37+7+190+273.
19. 188+560+108+506+56+15+7+180+18+56+566.
20. 673+840+737+928+517+349+210+500+618+819.
21. 307+509+910+117+250+638+356+951+117+89
22. 15+27+119+94+101+709+364+87+2+370+241.
23. 293+18+573+194+346+504+673+936+19+207.
24. 64+604+406+600+640+460+46+83+803+830.
25. 199+96+737+307+516+93+7+16+738+259+59.

20. EXERCISE X.

1.[1]	2.	3.	4.	5.	6.	7.	8.	9.
1,000	1000	5000	7000	1896	4567	8456	2408	9406
2,000	1100	500	700	1304	8432	7349	5493	1250
4,000	1200	4000	70	1940	9064	9118	9621	6430
6,000	1300	40	7	1284	2345	2565	8504	8094
8,000	1400	800	600	1700	7298	3894	7632	5432
9,000	1500	9000	4000	1676	5934	5248	4562	8006
7,000	1600	5000	900	1864	6309	7348	3901	9210
5,000	1700	600	6000	1547	7124	9176	2008	5090

10.	11.	12.	13.	14.	15.	16.	17.
3476	2930	8046	10,000	30,000	70000	80000	27,300
593	456	810	30,000	40,000	30000	500	34,000
24	3948	9	50,000	70,000	6000	60	26,900
896	27	9421	90,000	80,000	200	50000	84,200
7208	639	39	80,000	10,000	8000	9000	53,700
5009	7204	840	40,000	30,000	90000	40	85,600
648	408	7240	20,000	60,000	600	30000	28,400
8	3072	384	50,000	50,000	50000	700	61,060

[1] See Note, Ex. viii. p. 7.

18.	19.	20.	21.	22.	23.	24.
43,214	73059	83426	29070	45623	82472	19465
28,970	84320	34924	50846	72020	846	3947
36,429	92000	85241	63147	93647	9701	64
82,456	84372	12345	94621	804	35624	94702
93,484	50028	66666	80403	9562	256	876
21,086	90200	93002	70002	93	7	5724
73,481	89301	47020	70020	84756	9470	12730
18,498	56238	13076	70200	7250	85064	9400

25.	26.	27.	28.	29.	30.
100,000	300,000	400000	648,724	910,317	542300
300,000	200,000	8000	720,720	843,256	272484
700,000	700,000	90	843,843	123,000	364862
800,000	60,000	900	920,000	456,700	127859
400,000	50,000	9000	647,000	506,840	730640
900,000	500	80000	564,300	920,100	827938
500,000	800,000	800000	734,310	800,701	910400
600,000	500,000	60000	173,094	308,452	478915

31. 843+2465+724+17+10934+59470+107+20094+800.
32. 927+250+3070+601+38+731+1456+1001+27+374.
33. 493+913+67+500+610+1100+1420+3706+3076+3760.
34. 39+280+563+730+525+3482+79+2496+7314+326+89.
35. 470+1493+293+674+825+300+93+1910+2564+836+932.
36. 9246+29805+367934+39+493+9+90+49321+7007.
37. 8439+7246+297+800+2094+73825+493+12345+936.
38. 4731+8472+938+76+3938+425+18+967+2005+6790.
39. 4901+829+736+90+894+3247+9694+8482+386.
40. 7000+770+9382+54+504+5004+5040+5400+7054.
41. 348+7+77+777+7777+77777+9+49+17248+34.
42. 2693+301+4+404+39456+327+999+45602+18.
43. 24962+37642+4936+2754+930+18500+2590+196.
44. 93642+80010+930+18275+60600+66000+60060.
45. 7285+93271+893+7249+90000+18506+375+9640.
46. 8546+2764+94681+27600+9300+71486+8206+9.
47. 45894+318+7462+80001+90309+7402+70906.
48. 437+938+94+7300+1805+72468+79005+9406+50.
49. 6293+946+8001+92465+716+24070+807+5005+397.
50. 5484+29367+937056+720000+804906+100000+9040.
51. 249356+730854+272494+800800+549304+20400+701.
52. 42836+90045+89362+5279+7264+7649+1200+937.
53. 5000+50000+50+505+5050+5+555+55555+550.

EXERCISE XI.

Below the sum of the following numbers, write the *uppermost*, and add again; below that sum write the *second* from the top, and add again; continue the addition in this way till *all* the numbers are taken in, and find the sum.

1. 235+196+450+600+801.
2. 342+94+502+86+300.
3. 279+50+116+270+207.
4. 100+50+322+901+626.
5. 736+941+257+509+316.
6. 241+80+173+428+299.
7. 864+731+279+338+67.
8. 420+204+176+815+700.
9. 304+430+82+73+371.
10. 536+801+78+306+420.
11. 216+39+500+493+811.
12. 340+610+93+217+536.
13. 117+711+270+207+453.
14. 820+304+916+732+564.
15. 936+576+429+827+517.
16. 320+600+66+308+201.
17. 524+47+39+809+468.
18. 279+320+809+543+397.

21.

EXERCISE XII.

1.	2.	3.	4.	5.
238946	900500	1,000,000	8000000	3,564,236
72400	2736	3,000,000	800000	2,564,304
930	93	8,000,000	80000	2,197,629
645046	84293	4,000,000	8000	8,469,038
8434	701856	6,000,000	800	7,382,093
67	73900	7,000,000	80	2,946,904
93248	2784	9,000,000	90000	3,842,460
100484	932048	2,000,000	7000000	8,080,808

6.	7.	8.	9.	10.
3456729	9203564	37,240,000	72,483,624	193,700,070
3040506	964333	93,280,000	8,734,724	270,937,000
3004005	728	87,200,400	9,328	384,256,070
3000400	92100	93,400,860	904,374	930,184,293
2790364	8056720	85,085,023	87,208,936	127,249,130
8710800	5296	62,473,908	97,318	147,234,876
5623938	931724	24,084,573	9,438,729	310,249,364
7708804	8403208	16,946,004	47,082,970	172,849,564

11. 1234567+7238049+3947246+8420800+9220000.
12. 8004930+12340+7248436+9436+87+72456+9384567.
13. 72483624+8734724+9328+904374+87208936.
14. 27007070+2700707+94302+734+85693+9438729.
15. 37248734+946432+87324+9256491+80724300.
16. 125000890+700700700+193299870+240019000.
17. 738456938+248724807+301234563+384965724.
18. 2000000+7304524+5428946+7289476+180050+72004.
19. 47849562+93859627+2507923+804974+2904+93006.
20. 192496924+534920815+8256293+79000600+180000018.

EXERCISE XIII.

1. John has 38 marbles; he buys 20 more, wins 17, and gets 11 from a friend. How many has he now?

2. In a school, the first class has 15 scholars, the second 24, the third 27, the fourth 30, and the fifth 31. How many scholars are in the school?

3. If I pay 8 shillings for bread, 14 shillings for tea, 7 shillings for sugar, and 11 shillings for butter and cheese; how many shillings do I pay?

4. In a wood there are 41 oak-trees, 18 firs, 63 beeches, and 9 elms. How many trees in all?

5. A traveller went 110 miles by train, 62 miles by steamer, 17 miles by coach, and then he had to walk 2 miles. What was the length of his journey?

6. England has 52 counties, Scotland 33, and Ireland 32. How many counties in the whole?

7. A class of 26 pupils receives 14 new ones. How many pupils has it now?

8. Three apple-trees in a garden were shaken for fruit: if one gave 516 apples, and the other two 620 each, how many apples did they give in all?

9. Three omnibuses started on a pleasure-trip: one carried 23 persons, the second 32, and the third 26. If 4 were taken up by the way, how many persons were there in the party?

10. A grocer pays £140 for shop rent, £37 for taxes, £11 for rent of cellars, and he spends £75 on repairs. What is the whole expense?

11. In a railway train there were 79 first-class passengers, 101 second-class, and 249 third-class. How many passengers in all?

12. When will a boy born in 1855 be 69 years old?

13. From Glasgow to Stirling is 30 miles, from Stirling to Perth 31, from Perth to Aberdeen 90. How far from Glasgow to Aberdeen?

14. A merchant owes to one creditor £4275, to a second £531, to a third £300, and to a fourth £3005. How much does he owe?

15. A basket of eggs contains 232, another contains 35 more than the first, and a third 101 more than the second. How many eggs in all?

*** Only a few problems of the very simplest kind are presented at this stage: the pupil will be able to continue them to more advantage when he has learnt the four elementary rules. See Ex. § 55.

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23. SUBTRACTION.

Ex.—Of 689 trees in a park, 327 were cut down. How many remained standing?

Here we have to find the difference between two given numbers, or what remains when the less is taken from the greater.

The greater of the two numbers is called the *Minuend*, which means the number to be diminished; the less is called the *Subtrahend*, which means the number to be taken away.

The number which remains is called the *Difference* or *Remainder*.

The process of finding it is *Subtraction;* called, when the things are of one kind, as here, *Simple Subtraction.*

The *sign* of Subtraction is — (minus); thus 2 — 1 is 1.

We cannot find the difference between 689 and 327 at once; the numbers are too large. We must, therefore, subtract them *in parts;* for which purpose we must learn the subtraction of the first nine numbers.

Subtraction Table.

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
2 is 1	3 is 1	4 is 1	5 is 1	6 is 1	7 is 1	8 is 1	9 is 1	10 is 1
3 ... 2	4 ... 2	5 ... 2	6 ... 2	7 ... 2	8 ... 2	9 ... 2	10 ... 2	11 ... 2
4 ... 3	5 ... 3	6 ... 3	7 ... 3	8 ... 3	9 ... 3	10 ... 3	11 ... 3	12 ... 3
5 ... 4	6 ... 4	7 ... 4	8 ... 4	9 ... 4	10 ... 4	11 ... 4	12 .. 4	13 ... 4
6 ... 5	7 ... 5	8 ... 5	9 ... 5	10 ... 5	11 ... 5	12 ... 5	13 ... 5	14 ... 5
7 ... 6	8 ... 6	9 ... 6	10 ... 6	11 ... 6	12 ... 6	13 ... 6	14 ... 6	15 ... 6
8 ... 7	9 ... 7	10 ... 7	11 ... 7	12 ... 7	13 ... 7	14 ... 7	15 ... 7	16 ... 7
9 ... 8	10 ... 8	11 ... 8	12 ... 8	13 ... 8	14 ... 8	15 ... 8	16 ... 8	17 ... 8
10 ... 9	11 ... 9	12 ... 9	13 ... 9	14 ... 9	15 ... 9	16 ... 9	17 ... 9	18 ... 9

Bf.

EXERCISE I.

1. Repeat the several columns—backwards—by odds—by evens.
2. Subtract the units in each column from its highest number.
3. 3 from 8 leaves —? 4 from 13 leaves —? etc.
4. 9 less 2 less 3 is —? 17−8−4 is —? etc.
5. To 7 add 3 and take away 4? 9+8−2−2 is —?
6. From 5 books take 2, and how many remain? John had 6 marbles; if he lost 3 and then 1, how many had he? Jane has 7 pence; if she gets 6 pence more and gives away fourpence, what has she now? etc.
7. Write down the columns of the Table in order.

24. Subtraction of Numbers of Two Places.

The Table given above serves also for the subtraction of tens; thus:—

If 1 from 2 is 1, 1 ten from 2 tens is 1 ten, or 10 from 20 is 10.
If 1 from 3 is 2, 1 ten from 3 tens, is 2 tens, or 10 from 30 is 20.
Etc. etc. etc.
If 2 from 3 is 1, 2 tens from 3 tens is 1 ten, or 20 from 30 is 10.
If 2 from 4 is 2, 2 tens from 4 tens is 2 tens, or 20 from 40 is 20.
Etc. etc. etc.

EXERCISE II.

Perform Ex. i. with tens instead of units.

Ex.—A woman had 76 eggs in a basket; if she sold 34, how many had she remaining?

Set down the subtrahend below the minuend *in its place;* then, subtract the places *in their order.*	76 34
4 from 6 is 2 *units;* set down the 2 in its place.	—
3 from 7 is 4 *tens;* set down the 4 in its place.	42

Total difference, 42.

To prove the result, add together the subtrahend and the difference; the sum should be the minuend, since what is taken away from a number and what is left of it make up between them the whole number.

EXERCISE III.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
84	56	76	48	59	37	29	70	86	91	64	73
32	24	36	25	32	21	19	30	20	31	20	52

13. 47 − 24 16. 39 − 19 19. 81 − 41 22. 85 − 42
14. 78 − 51 17. 40 − 20 20. 56 − 36 23. 71 − 31
15. 63 − 30 18. 93 − 63 21. 78 − 47 24. 99 − 57

25. Though the minuend must always be greater than the subtrahend, any place of the minuend except the highest may be less than the place below it of the subtrahend.

Ex.—A teacher has 45 steel pens; if he distributes 29 to his class, how many are over?

9 from 5 cannot be taken; change one of the tens into units, making 15 units in all; 9 from 15 is 6 units, set down the 6 in its place.	45
2 from 3 (the 3 tens remaining) is 1 ten; set down the 1 in its place.	29
Total difference, 16.	16

Rule.—Write the less number under the greater *in its place*; subtract the columns in their order beginning with the units'; change one of the next highest name when necessary.

Or thus,[1]

9 from 5 cannot be taken; add 10 *units* to the 5, making 15 in all; 9 from 15 is 6 units.	45
Add 1 *ten* to the 2 tens; 3 from 4 is 1 *ten*.	29
Total difference, as before, 16.	16

In adding 10 *units* to the minuend and 1 *ten* to the subtrahend, we have added *the same number* to both. This does not alter their difference; but makes it easier to find, by keeping each place of the minuend greater than the place below it of the subtrahend.

Rule.—Write the less number under the greater *in its place*; subtract the columns in their order beginning with the units'; add *ten* to any place of the minuend which is less than the place below it of the subtrahend, and one to the *next* place of the subtrahend.

[1] Both methods of subtraction are given; the teacher may choose either.

EXERCISE IV.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
a.	35	47	53	64	71	60	82	91	47	24	63	30	44	28	34	41
	17	39	27	35	49	29	35	53	19	17	45	21	27	9	16	27
	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	31.	32.
b.	44	21	43	94	42	76	48	32	51	36	22	74	52	81	34	45
	18	12	24	47	25	39	29	17	37	17	13	49	26	39	27	19
	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	84.
c.	53	70	42	80	48	30	52	63	74	85	96	97	50	32	43	77
	18	43	19	37	19	12	24	34	45	67	29	38	26	27	17	58

49. 66 – 17	57. 21 – 13	65. 74 – 16
50. 47 – 28	58. 38 – 19	66. 81 – 25
51. 23 – 14	59. 72 – 43	67. 62 – 37
52. 55 – 27	60. 83 – 54	68. 50 – 23
53. 70 – 34	61. 51 – 26	69. 27 – 9
54. 84 – 27	62. 66 – 37	70. 34 – 15
55. 95 – 46	63. 80 – 43	71. 53 – 27
56. 60 – 24	64. 91 – 54	72. 67 – 39

73. 44 – 27
74. 58 – 39
75. 86 – 48
76. 90 – 54
77. 93 – 65
78. 45 – 29
79. 74 – 36
80. 82 – 43

EXERCISE V.

Perform the above exercise mentally.

*** In doing so, it is more convenient to subtract the tens first, and then the units ; thus in 35 – 17, 10 from 35 leaves 25, and 7 from 25 leaves 18.

26. Subtraction of Numbers of One or more Periods.

The Table given, sect. 23, serves also for the subtraction of hundreds, thousands, &c. ; thus :

If 1 from 2 is 1, 1 hund. from 2 hund. is 1 hund., or 100 from 200 is 100.
If 1 from 3 is 2, 1 hund. from 3 hund. is 2 hund., or 100 from 300 is 200.
Etc. etc. etc.
If 2 from 3 is 1, 2 hund. from 3 hund. is 1 hund., or 200 from 300 is 100.
If 2 from 4 is 2, 2 hund. from 4 hund. is 2 hund., or 200 from 400 is 200.
Etc. etc. etc.

EXERCISE VI.

Perform Ex. i. with hundreds instead of units.

Ex. 1. Of 689 trees in a park, 327 were cut down : how many remained standing ?

7 from 9 is 2 units ; set down the 2 in its place.	689
2 from 8 is 6 tens ; set down the 6 in its place.	327
3 from 6 is 3 hund. ; set down the 3 in its place.	———
Total difference, 362.	362

Ex. 2. How much greater is 6073 than 484?

In this example, there is a cipher in the minuend, and the highest place of the minuend has no place below it in the subtrahend.

4 from 13 is 9 for the units' place.
8 from 16 (changing one of the next highest name, which is thousands) is 8 for the tens' place.
4 from 9 (the 9 hundreds remaining when the one thousand was changed) is 5 for the hundreds' place.
0 from 5 is 5 for the thousands' place.

6073
484
5589

Or thus:

4 from 13 is 9 for the units' place.
9 from 17 is 8 for the tens' place.
5 from 10 is 5 for the hundreds' place.
1 from 6 is 5 for the thousands' place.

27.

EXERCISE VII.

1.	2.	3.	4.	5.	6.	7.	8.	9.
796	805	909	483	857	564	769	960	637
454	403	100	150	724	203	456	500	415

10. 758 − 342
11. 975 − 600
12. 856 − 326
13. 576 − 420
14. 874 − 574
15. 716 − 516
16. 7345 − 5135
17. 8500 − 7000
18. 2021 − 1020

EXERCISE VIII.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
547	635	248	429	511	924	700	801	540	707	800	600
219	427	154	274	364	519	451	605	229	593	209	405

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
713	391	420	706	300	401	535	297	316	802	732	194
256	98	301	279	107	208	328	198	49	541	342	94

25.	26.	27.	28.	29.	30.	31.	32.	33.	34.
3429	5642	7396	4524	8527	5418	4000	5040	6080	9004
2763	3804	5409	2790	6050	2980	2534	3956	2500	5084

35.	36.	37.	38.	39.	40.	41.	42.	43.	44.
7320	8074	2094	7000	5484	9302	7549	1368	2008	7309
2496	1943	859	3456	2390	1903	5840	509	1123	978

45. 407 – 298
46. 630 – 450
47. 275 – 87
48. 116 – 58
49. 730 – 563
50. 805 – 496
51. 357 – 192
52. 207 – 84
53. 476 – 189
54. 520 – 218
55. 600 – 315
56. 2809 – 939
57. 7340 – 2093
58. 9008 – 572
59. 1009 – 450
60. 7084 – 3921
61. 8000 – 1090
62. 5009 – 3094
63. 9101 – 9011
64. 7308 – 5904
65. 8234 – 4731
66. 2890 – 1936

EXERCISE IX.

*** In the following, find the first remainder less than the subtrahend.

1. 402 – 86
2. 530 – 105
3. 736 – 209
4. 900 – 121
5. 437 – 99
6. 215 – 67
7. 600 – 143
8. 816 – 197
9. 701 – 156
10. 2760 – 672
11. 8207 – 1938
12. 6094 – 856
13. 9400 – 2763
14. 8405 – 1504
15. 3091 – 750
16. 7463 – 1976
17. 5000 – 987
18. 5185 – 1978
19. 7320 – 2094
20. 9017 – 1853

28.

EXERCISE X.

1.	2.	3.	4.	5.	6.	7.
45060	38905	27936	84571	73021	45239	84901
29360	19450	10007	25038	49950	29308	56402

8.	9.	10.	11.	12.	13.
378923	934856	734085	400000	501020	276408
194033	256094	508506	40401	392406	120394

14.	15.	16.	17.	18.
2567283	45070134	23900140	50000014	100200300
730946	29098040	4015002	6010305	100199025

19. 25678 – 19341
20. 38056 – 9456
21. 45804 – 993
22. 50600 – 5600
23. 89476 – 4890
24. 793246 – 45600
25. 840300 – 524080
26. 608409 – 93560
27. 900000 – 90909
28. 257931 – 80002
29. 456890 – 193456
30. 8409302 – 908567
31. 10000000 – 1001001
32. 57340506 – 8530205
33. 73894219 – 25934764
34. 170170170 – 7107100
35. 59340947 – 20560724
36. 123456789 – 98764532
37. 10000000 – 100000
38. 500500500 – 650650

*** In the following, find the first remainder less than the subtrahend.

39. 56030 – 9807
40. 10101 – 3427
41. 27092 – 5083
42. 47138 – 7509
43. 60930 – 9493
44. 127936 – 29647
45. 982401 – 109472
46. 273408 – 84279
47. 730294 – 165085
48. 100901 – 10192
49. 605090 – 92071
50. 400000 – 101010

29.

EXERCISE XI.

1. Count back by twos from 100, from 101.
2. Count back by threes from 102, from 101, from 100.
3. Count back by fours from 100, from 101, from 102, from 103.
4. Count back by fives from 100, 101, 102, 103, 104.
5. Count back by sixes from 102, 101, 100, 103, 104, 105.
6. Count back by sevens from 105, 104, 103, 102, 101, 100, 106.
7. Count back by eights from 100, 101, 102, 103, 104, 105, 106, 107.
8. Count back by nines from 108, 107, 106, 105, 104, 103, 102, 101.

*** This and the following Ex. should be practised along with the foregoing.

EXERCISE XII.

1. 3+2+9−5−4+1−3+8+2−7−1+3+6−4−5+5+6−8.
2. 7+4−3+5+7−5+9+6−7−3+9+4−9−1+8+5−7−3.
3. 15−8+9−4+9+5−3−7+4−5+10+20−11−7+4+8−5−10.
4. 22+8−11−4+8+4−7−1+9+4−3+2−7+9−4−3+6+7−5.
5. 40+3−7−4+20−7−9−4+8−7+6−8+10−5−7−2+8+10.
6. 14+3−9+10−6−2+20−5+9−7−10+11+11−8+13−4−12.
7. 36+9−4−8+2+5+9−12−10+7+9−4−8+5+4−6−8+20.
8. 19+9−5−8+7+10−5−11+20−11+9+4−5+5−4+9−7−4.
9. 28+10+7−12−10+6−7−4+3+1−9−8+11+5−7−9+8+4.
10. 50+10−20+30+16−10−10+20+7−10+20+50−30−7−10.
11. 49−5+12−9+16−10+8+13−5−8−7+11+7+15−30−10+9.
12. 53+8−11+5+9−14−15+9+30−4−9+12+20−5+16−13−9.

Etc. etc. etc.

EXERCISE XIII.

How many are 37−29+48−33+79−15?

Here, instead of subtracting 29 from 37, then adding 48, and so on, it is shorter to add together the numbers which are +, then add together the numbers which are −, and find the difference of the two sums, thus:—

37	−29	For it is the same thing whether, in finding 9−2−2, we say 2 from 9 is 7, 2 from 7 is 5; or 2 and 2 are 4, 4 from 9 is 5.
+48	−33	
+79	−15	
164	−77 is 87	

1. 125+37−84−10+76+53−101+56+279−184−45+293.
2. 74−40+51−9+29+16−19−5+36−27+40+11.
3. 18+15−10+40+36−19−14+23−39+20+16−19.
4. 56+20−43−27+39+24−31+64−45+21+10−34.
5. 90+45+16−49−51+6−15+39−60+49+53··19.
6. 36−19+53−29+36−24−11−9+64+17−24−9+14.
7. 49+36−29−14+20+36−18−9+25+84−59−27+40.
8. 74+52−63−10+29+37−45−37+22−51+69−19+26.
9. 192−56−14+58+213−191+64−49+346−154−48+90.
10. 724−593+824−48+93+702−500+293−59−73+256−100.
11. 50004−8456−401+4592+9400−10100+734−809.
12. 29340−4560−9390+7248−15600+93402−56840.

30. EXERCISE XIV.

1. A woman went to market with a basket of eggs containing 342: if she sold 192, how many did she bring back?
2. John has 95 nuts, but gives 37 to William. How many does he keep?
3. A teacher gives out pens to a class of 60 scholars, but the box has only 37. How many does he want?
4. A cheese weighs 78 pounds. How much heavier is it than another which weighs only 47 pounds?
5. A tradesman owes £260, but he has only £137. How much does he require to pay his debts?

6. A cask of sugar contains 539 pounds' weight. How much must be sold to leave 257 pounds?

7. James has 24 marbles, and his brother gives him 37. How many must he buy to make up 100?

8. If a school has 374 scholars, of whom 27 are in the first class, and 32 in the second; how many are in the other classes together?

9. A green-grocer received a basket of apples and pears, 264 in all: 157 were apples; how many were pears?

10. A house is worth £520, but it will cost £84 to repair it. How much should it be sold for?

11. Edinburgh to Dunbar is 29 miles, and Edinburgh to Berwick is 57 miles. How far from Dunbar to Berwick?

12. A tradesman earns 16s. a week, and spends 13s. How much does he save in four weeks?

13. A farmer had in his yard 31 fowls, 17 geese, 24 turkeys, and his ducks made up the entire number of his poultry to 87. How many ducks had he?

14. How much of 385 yards remains if 93 yards be cut away from the piece? How often may 93 yards be cut away, and what will remain?

15. A train started with 374 passengers. At the first station 16 went out and 9 came in; at the second, 11 went out and 25 came in; at the third, 3 went out. How many passengers left the train at the terminus?—See Ex. § 55.

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31. MULTIPLICATION.

Ex.—Five boxes of oranges contained 120 each, how many oranges were there in all?

Here we have to find a number equal to 120 repeated 5 times.

We could find that by adding 125 to itself 5 times; but a shorter way is to *multiply* 125 by 5.

The number to be repeated is called the *multiplicand.*
The number of times it is to be repeated, *multiplier.*
Both are sometimes called the . . *factors.*
The result is called the . . . *product.*

The process is called *multiplication;* and, when the multiplicand is of one kind as here, *simple multiplication.*

The sign of multiplication is × (*multiplied by*); thus 2 × 2 are 4.

We cannot find how much 5 times 125 is by one step; the multiplicand is too large. We must therefore do it *in parts;* for which purpose we must learn the multiplication of the first nine numbers.

32. Multiplication Table.

2 times			3 times			4 times			5 times			6 times			7 times		
1	are	2	1	are	3	1	are	4	1	are	5	1	are	6	1	are	7
2	...	4	2	...	6	2	...	8	2	...	10	2	...	12	2	...	14
3	...	6	3	...	9	3	...	12	3	...	15	3	...	18	3	...	21
4	...	8	4	...	12	4	...	16	4	...	20	4	...	24	4	...	28
5	...	10	5	...	15	5	...	20	5	...	25	5	...	30	5	...	35
6	...	12	6	...	18	6	...	24	6	...	30	6	...	36	6	...	42
7	...	14	7	...	21	7	...	28	7	...	35	7	...	42	7	...	49
8	...	16	8	...	24	8	...	32	8	...	40	8	...	48	8	...	56
9	...	18	9	...	27	9	...	36	9	...	45	9	...	54	9	...	63
10	...	20	10	...	30	10	...	40	10	...	50	10	...	60	10	...	70
11	...	22	11	...	33	11	...	44	11	...	55	11	...	66	11	...	77
12	...	24	12	...	36	12	...	48	12	...	60	12	...	72	12	...	84

8 times			9 times			10 times			11 times			12 times		
1	are	8	1	are	9	1	are	10	1	are	11	1	are	12
2	...	16	2	...	18	2	...	20	2	...	22	2	...	24
3	...	24	3	...	27	3	...	30	3	...	33	3	...	36
4	...	32	4	...	36	4	...	40	4	...	44	4	...	48
5	...	40	5	...	45	5	...	50	5	...	55	5	...	60
6	...	48	6	...	54	6	...	60	6	...	66	6	...	72
7	...	56	7	...	63	7	...	70	7	...	77	7	...	84
8	...	64	8	...	72	8	...	80	8	...	88	8	...	96
9	...	72	9	...	81	9	...	90	9	...	99	9	...	108
10	...	80	10	...	90	10	...	100	10	...	110	10	...	120
11	...	88	11	...	99	11	...	110	11	...	121	11	...	132
12	...	96	12	...	108	12	...	120	12	...	132	12	...	144

*** This Table should be learnt first in lines even along, then in lines up and down. The pupil should practise it daily till he has it thoroughly at command.

EXERCISE. I. *Bf.*

1. Repeat the several lines even along; backwards; by odds; by evens.
2. Repeat the lines up and down; backwards; by odds; by evens.
3. 4 times 5 are — ? 6 times 9 are — ? 8 times 7 are — ? etc.
   5 times 4 are — ? 9 times 6 are — ? 7 times 8 are — ? etc.
4. How many fingers have 8 boys? How many wheels have 9 carts? How many days have seven weeks? How many farthings have four pence? How many units in 5 tens? How many marbles have 9 boys with 11 each? What cost 6 oranges at 2 pence each? 7 fowls at 3 shillings each? etc.
5. Name two factors of 18, 24, 96, etc.
6. How many times 7 is 63? 21? 70? etc.
7. 36 is 9 times — ? 72 is 6 times — ? etc.
8. 3 times 6 + 2 are — ? 5 times 8 with 9 added are — ? etc.
   4 times 12 less 9 are — ? 7 times 5 − 6 are — ? etc.
9. 2 times 4 and 3 times that are — ? etc.
   6 multiplied twice by 2 are — ? etc.
10. Write down the several columns of the Table.

33. The Table given above serves also for the multiplication of tens, hundreds, etc. Thus—

If 2 times 1 are 2, 2 times 1 ten are 2 tens, or 2 times 10 are 20.
If 2 times 2 are 4, 2 times 2 tens are 4 tens, or 2 times 20 are 40.
Etc. etc. etc.
If 3 times 3 are 9, 3 times 3 tens are 9 tens, or 3 times 30 are 90.
Etc. etc. etc.

EXERCISE II.

Perform Ex. i. with tens in the multiplicand.

If 2 times 1 are 2, 2 times 1 h. are 2 h., or 2 times 100 are 200.
If 2 times 2 are 4, 2 times 2 h. are 4 h., or 2 times 200 are 400.
Etc. etc. etc.

EXERCISE III.

Perform Ex. i. with hundreds in the multiplicand.

Multiplication by Units.

34. *Ex.*—Five boxes of oranges contained 125 each, how many oranges were there in all?

Set the multiplier below the multiplicand in its place; then, multiplying each place in its order,

5 times 5 are 25 units; set down 5 units and carry 2 tens. 125
5 times 2 are 10, and 2 are 12 tens; set down 2 tens and carry 1 hundred. 5
5 times 1 are 5, and 1 are 6 hundreds. 625
Product, 625.

Rule.—To multiply by units, multiply each place of the multiplicand in order, carrying tens.

The answer may be proved by adding the multiplicand to itself 5 times; the sum should be the same as the product. Or we may multiply by 4, the number next below the multiplier, and add the multiplicand to the product.

EXERCISE IV.

1. Multiply the following numbers by 2, 3, etc., to 12, in order :—

13	21	31	41	51	61	71	81	91
14	22	32	42	52	62	72	82	92
15	23	33	43	53	63	73	83	93
16	24	34	44	54	64	74	84	94
17	25	35	45	55	65	75	85	95
18	26	36	46	56	66	76	86	96
19	27	37	47	57	67	77	87	97
20	28	38	48	58	68	78	88	98
	29	39	49	59	69	79	89	99
	30	40	50	60	70	80	90	100

2. Multiply the several columns mentally.
3. 2 times 27 are — ? 3 times 32 are — ? 4 times 48 are — ? etc.
4. Multiply the following numbers by 2, 3, etc., to 12, in order :—

1. 111	11. 893	21. 2461	31. 24682	41. 34194
2. 222	12. 248	22. 5382	32. 74394	42. 21384
3. 333	13. 604	23. 2081	33. 31208	43. 75689
4. 444	14. 573	24. 4095	34. 24295	44. 38472
5. 555	15. 421	25. 2496	35. 19064	45. 29319
6. 666	16. 298	26. 5162	36. 70538	46. 82964
7. 777	17. 157	27. 7349	37. 25819	47. 70109
8. 888	18. 820	28. 8210	38. 39147	48. 10840
9. 999	19. 659	29. 9347	39. 16731	49. 30028
10. 427	20. 416	30. 1924	40. 42858	50. 90084

*** This exercise is designed to be performed orally from the book as well as on slate.

35. Multiplication by Factors.

Ex.—Multiply 248 by 24.

Since 24 is 6 times 4, we multiply by 24, if we multiply first by 6, and then that product by 4 ; thus :—

```
 248
   6
------
1488
   4
------
5952
```

The result may be proved by multiplying by 3 and 8, or by 2 and 12 ; which are also factors of 24, and which should therefore give the same product.

A number like 24 which is made up of factors (other than 1) is called a *composite number.*

A number like 7, 11, or 23, which is not made up of factors, is called a *prime number.*

Multiplication by two factors may be used in the case of all composite multipliers between 12 and 144.

Practice in multiplying will show the pupil that three factors may often be used for a multiplier with advantage ; thus, $252=4\times7\times9$.

EXERCISE V.

Multiply, using factors :—

1. 536×14, 15, 21, 22.
2. 270×25, 27, 28, 32.
3. 905×33, 42, 44, 45.
4. 827×54, 55, 56, 63.
5. 638×63, 66, 72.
6. 4732×77, 81, 84.
7. 2096×88, 96, 99.
8. 8405×108, 121, 132.
9. 7289×144, 160, 270.
10. 8175×420, 840.
11. 3497×16, 18, 48, 72, in two ways.
12. 7302×24, 36, in three ways.

36. Multiplication by more than One Place.

A cipher annexed to the right of a figure increases its value 10 times, that is, multiplies it by 10. Therefore, to multiply by 2 tens or 20, multiply by 2, and annex the cipher; to multiply by 30, multiply by 3, and annex the cipher; and so on.

Similarly to multiply by 200, multiply by 2, and annex two ciphers; to multiply by 300, multiply by 3, and annex two ciphers; and so on.

Rule.—To multiply by tens, hundreds, etc., multiply by the left-hand figure, and annex the ciphers.

EXERCISE VI.

1. Multiply the columns in Ex. iv. by 20, 40, 50, 90.
2. Multiply the same columns by 300, 600, 700, 800.

37.

Ex.—A book contains 356 pages, and each page 237 words: how many words are in the book?

Set the multiplier below the multiplicand in its place; then multiplying by the 7 units, we have	356 237 2492
Multiplying by the 3 tens, we have . .	10680
Multiplying by the 2 hundreds, we have .	71200
Product by whole mutiplier is . .	84372

The result may be proved by interchanging the multiplier and multiplicand, that is, multiplying 237 by 356; which will give the same product.

Rule.—To multiply by a number of several places, multiply by each place in order from the units, and add the several products.

*** The pupil may by and by omit the ciphers, denoting the tens and hundreds in the second and third lines of multiplication; being careful to place the right-hand figure of each line exactly under that place of the multiplier which gives it.

Should there be a cipher in the tens, or some higher place in the multiplier, it is simply passed over in multiplying.

EXERCISE VII.

1. 2364×29, 37, 43.
2. 4328×39, 51, 86.
3. 5936×28, 46, 59.
4. 9320×19, 73, 31.
5. 8256×17, 93, 49.
6. 6439×38, 57, 61.
7. 20480×71, 43, 53.
8. 30093×98, 83, 78.
9. 40001×81, 28, 34.
10. 73000×47, 59, 92.
11. 90000×27, 64, 79.
12. 70091×75, 88, 99.

EXERCISE VIII.

1. 35627×183, 297, 403.
2. 47231×245, 318, 721.
3. 93086×240, 825, 649.
4. 23456×409, 207, 308.
5. 73610×930, 470, 290.
6. 85093×418, 738, 562.
7. 72170×936, 259, 816.
8. 37293×904, 506, 801.
9. 80050×629, 350, 680.
10. 90000×456, 789, 910.
11. 70700×843, 529, 365.
12. 90280×706, 504, 209.
13. 456789×297, 399, 536.
14. 724936×840, 908, 273.
15. 459630×364, 814, 518.
16. 536298×230, 563, 720.
17. 210830×821, 913, 713.
18. 914567×439, 546, 208.

EXERCISE IX.

1. 500606×5423, 6106.
2. 730000×2936, 8492.
3. 700000×4028, 5003.
4. 830830×6300, 7240.
5. 308070×8740, 5007.
6. 934764×23418, 93125.
7. 621930×19728, 73465.
8. 493628×27368, 93480.
9. 840300×19030, 80807.
10. 621934×70029, 54309.
11. 493002×56721, 12765.
12. 2389745×4567, 7394, 6270.
13. 6348576×7321, 8492, 1029.
14. 2930840×6080, 5090, 7200.
15. 7394900×8936, 2009, 5900.
16. 8002006×7290, 5718, 3290.
17. 7802058×35467, 29631.
18. 4932096×84932, 94629.
19. 7007007×93021, 80709.
20. 3489493×29100, 28101.
21. 9000000×73506, 82090.
22. 4290000×80972, 50608.

EXERCISE X.

1. 25473809×258956, 817456.
2. 73890496×483921, 293185.
3. 90900900×259671, 798491.
4. 25608709×408506, 930850.
5. 70409360×273093, 129608.
6. 49328914×506090, 709080.
7. 82483949×210000, 930039.
8. 72340090×724801, 520936.
9. 53042485×493094, 891172.
10. 73249000×938950, 249056.
11. 490562001×362987, 450893.
12. 293904510×450813, 920854.
13. 710842930×293050, 493096.
14. 256849361×259928, 936190.
15. 209209209×123456, 789012.
16. 600040068×900405, 908550.
17. 394620100×736493, 856190.
18. 824904561×437285, 737292.
19. 296382173×555555, 505050.
20. 493084095×828561, 400800.

38. Squares and Cubes.

A figure like this, which has 4 rows of counters, each containing 4, is called a *square*. The number of counters we see by counting to be 16; that is, the number even along (4) multiplied by the number up and down (4). *Bf.*

. . . .
. . . .
. . . .
. . . .

Similarly 7 rows of trees with 7 trees in each would be a square of 49; 10 lines of soldiers with 10 soldiers in each line would be a square of 100.

When any number is multiplied by itself, the product is called the *square* or *second power* of that number. The square of 4 is denoted 4^2.

EXERCISE XI.

1. Repeat the squares of 1, 2, 3, 4, etc., up to 12.
2. Find the squares of 13, 14, 15, 16, 17, 18, 19, 20.
3. Find the squares of these numbers :—

1. 784	5. 3456	9. 23456	13. 75423	17. 50005
2. 937	6. 2930	10. 90307	14. 20056	18. 728946
3. 508	7. 4500	11. 58126	15. 90030	19. 809407
4. 610	8. 7000	12. 37000	16. 80705	20. 916738

39. When a number is multiplied twice by itself, the product is called the *cube* or *third power* of that number ; thus $4\times4\times4=64$. The cube of 4 is denoted 4^3.

*** This may be illustrated by a small cube of wood, or, better still, by a box of such cubes.

EXERCISE XII.

1. What are the cubes of 1, 2, 3, etc., up to 10?
2. Find the cubes of these numbers :—

1. 789	4. 4506	7. 12000	10. 67809
2. 405	5. 5730	8. 37100	11. 40506
3. 628	6. 9825	9. 24089	12. 12345

40.

EXERCISE XIII.

1. How many eggs in 16 boxes, each having 96?
2. How many pupils in a school which has 7 classes of 23 each?
3. How many hours in 36 days?
4. How many pence in 47 half-crowns?
5. How many oranges, at 15 for a shilling, will 25s. buy?
6. How long a journey shall I make in 27 days, at 18 miles a day?
7. How many yards of linen in 387 pieces, each 35 yards?
8. How many bottles in 45 dozen and 5?
9. How many pages in a yearly volume, of which a monthly part has 96?
10. What cost a railway 49 miles long, at £4500 a mile?
11. A postman delivers 29 letters each morning and evening for a week ; how many did he deliver in all?
12. A pipe pours into a cistern daily 13410 gallons water ; how many gallons will it pour in during November?
13. A house of five storeys has seven windows in each, and twelve panes of glass in each window ; how many panes of glass are there in all?
14. Three men, in business together, receive £672 each of the profits at the end of the first year ; what were the whole profits?
15. If a baker reckons 13 to a dozen, how many biscuits does he count to 136 dozen?
16. A merchant's office occupies 43 clerks at £2 a week each, and 24 at £3 ; what sum is required in a year for their wages?
17. There are 129 trees in the side of a square plantation ; how many trees has the plantation?

41. DIVISION.

Ex.—A box of eggs, containing 852, is to be divided amongst a number of families, each getting 6 ; how many families will be served ?

Here we have to find how often 6 is contained in 852.

We could find that by subtracting 6 from 852 successively till nothing remains, and then counting the number of 6's we have got, but a shorter way is to *divide* 852 by 6.

The number to be divided is called the *dividend.*

The dividing number is called the *divisor.*

The number of times the divisor is contained in the dividend is called the *quotient.*

The process of dividing is called *division ;* and, where the dividend is of one kind as here, *simple division.*

The sign of division is ÷ (*divided* by) ; thus, 4 ÷ 2 is 2.

We cannot find how often 9 is contained in 243 by one step ; the dividend is too large for that. We must therefore do it in parts, for which purpose we must learn the division of the first nine numbers.

42. Division Table.

2 in	3 in	4 in	5 in	6 in	7 in
2 is 1	3 is 1	4 is 1	5 is 1	6 is 1	7 is 1
4 ... 2	6 ... 2	8 ... 2	10 ... 2	12 ... 2	14 ... 2
6 ... 3	9 ... 3	12 ... 3	15 ... 3	18 ... 3	21 ... 3
8 ... 4	12 ... 4	16 ... 4	20 ... 4	24 ... 4	28 ... 4
10 ... 5	15 ... 5	20 ... 5	25 ... 5	30 ... 5	35 ... 5
12 ... 6	18 ... 6	24 ... 6	30 ... 6	36 ... 6	42 ... 6
14 ... 7	21 ... 7	28 ... 7	35 ... 7	42 ... 7	49 ... 7
16 ... 8	24 ... 8	32 ... 8	40 ... 8	48 ... 8	56 ... 8
18 ... 9	27 ... 9	36 ... 9	45 ... 9	54 ... 9	63 ... 9
20 ... 10	30 ... 10	40 ... 10	50 ... 10	60 ... 10	70 ... 10
22 ... 11	33 ... 11	44 ... 11	55 ... 11	66 ... 11	77 ... 11
24 ... 12	36 ... 12	48 ... 12	60 ... 12	72 ... 12	84 ... 12

8 in	9 in	10 in	11 in	12 in
8 is 1	9 is 1	10 is 1	11 is 1	12 is 1
16 ... 2	18 ... 2	20 ... 2	22 ... 2	24 ... 2
24 ... 3	27 ... 3	30 ... 3	33 ... 3	36 ... 3
32 ... 4	36 ... 4	40 ... 4	44 ... 4	48 ... 4
40 ... 5	45 ... 5	50 ... 5	55 ... 5	60 ... 5
48 ... 6	54 ... 6	60 ... 6	66 ... 6	72 ... 6
56 ... 7	63 ... 7	70 ... 7	77 ... 7	84 ... 7
64 ... 8	72 ... 8	80 ... 8	88 ... 8	96 ... 8
72 ... 9	81 ... 9	90 ... 9	99 ... 9	108 ... 9
80 ... 10	90 ... 10	100 ... 10	110 ... 10	120 ... 10
88 ... 11	99 ... 11	110 ... 11	121 ... 11	132 ... 11
96 ... 12	108 ... 12	120 ... 12	132 ... 12	144 ... 12

EXERCISE I. *Bf.*

1. Repeat the lines of this Table up-and-down; backwards; by odds; by evens.
2. Repeat the lines even along in the same way.
3. 2 in 8 is —? 5 in 35 is —? 9 in 72 is —? etc.
   4 in 8 is —? 7 in 35 is —? 8 in 72 is —? etc.
4. How many pence in 8 farthings? Divide 15 shillings among 5 persons. Divide 40 marbles among 8 boys. How many oranges at 2d. each can I buy with 16 pence? etc.
5. Write down the several columns of the Table.

43. This Table serves also for the division of tens, hundreds, etc. Thus—

If 2 in 2 is 1, 2 in 2 tens is 1 ten, or 2 in 20 is 10.
If 2 in 4 is 2, 2 in 4 tens is 2 tens, or 2 in 40 is 20.
Etc. etc. etc.
If 3 in 3 is 1, 3 in 3 tens is 1 ten, or 3 in 30 is 10.
Etc. etc. etc.

EXERCISE II.

Perform Ex. i., Nos. 1, 2, 3, with tens in the dividend.

If 2 in 2 is 1, 2 in 2 hunds. is 1 hund., or 2 in 200 is 100.
If 2 in 4 is 2, 2 in 4 hunds. is 2 hund., or 2 in 400 is 200.
Etc. etc. etc.

EXERCISE III.

Perform Ex. i., Nos. 1, 2, 3, with hundreds in the dividend.

44. Division by Numbers of One Place.

Ex.—How often is 3 contained in 963?
Place the divisor to the left of the dividend.
3 in 9 hundreds is 3 hundreds.
3 in 6 tens is 2 tens.
3 in 3 units is 1 unit.
Quotient, 321.

$$3 \overline{)\,963}\quad 321$$

EXERCISE IV.

Divide—

1. By 2: 86, 128, 420, 642, 864, 4806, 6428.
2. By 3: 63, 96, 123, 249, 630, 963, 6093.
3. By 4: 84, 168, 244, 488, 804, 884, 4084.
4. By 5: 105, 155, 250, 355, 505, 4550, 5035.
5. By 6: 126, 246, 306, 426, 5460, 6048, 12660.
6. By 7: 147, 217, 357, 714, 6377, 7063.
7. By 8: 168, 248, 320, 880, 1608, 5680.
8. By 9: 189, 279, 540, 3609, 4599, 8190.

45. The places of the dividend do not often contain the divisor evenly; there is generally a remainder.

EXERCISE V.

2 in 3 is 1 and 1 over; in 5 is —? in 7 is —? etc.
3 in 4 is 1 and 1 over; in 5 is —? in 7 is —? etc.
4 in 5 is 1 and 1 over; in 6 is —? in 7 is —? etc.
5 in 6 is 1 and 1 over; in 7 is —? in 8 is —? etc.

*** The exercise should be continued up to 12 as divisor.

46. *Ex.* 2.—A box of eggs, containing 852, is to be divided amongst a number of families, each getting 6; how many families will be served?

Set the divisor to the left of the dividend. Then 6 in 8 hundreds is 1 hundred and 2 hundreds over; set down the 1 in its place, and change the 2 hundreds into tens, making 25 in all.

$$6\overline{)852}\quad 142$$

6 in 25 tens is 4 tens and 1 ten over; set down the 4 in its place, and change the 1 ten into units, making 12 in all.

6 in 12 units is 2 units.

Quotient, 142.

Rule.—To divide by a number of one place, divide the places of the dividend in order from the highest, carrying the tens.

The result may be proved by multiplying the quotient by the divisor; the product should be the dividend.

EXERCISE VI.

Divide

1. By 2: 98, 258, 374, 454, 526, 598, 638, 694, 738, 876, 938, 972.
2. By 3: 87, 378, 465, 471, 513, 582, 648, 657, 726, 735, 879, 978.
3. By 4: 96, 492, 536, 548, 620, 676, 768, 792, 860, 892, 948, 956.
4. By 5: 565, 590, 675, 680, 745, 775, 865, 880, 930, 975, 7345.
5. By 6: 150, 672, 726, 744, 804, 852, 918, 990, 6834, 8526, 8730.
6. By 7: 161, 798, 805, 875, 910, 987, 7847, 7952, 8596, 8764, 9233.
7. By 8: 256, 896, 960, 992, 8976, 9544, 1896, 1944, 2888, 3976.
8. By 9: 144, 252, 423, 603, 828, 1026, 2160, 3267, 5040, 6543, 7038.
9. By 10: 730, 840, 9320, 4500, 7310, 2030.
10. By 11: 748, 396, 594, 286, 7942, 8503, 25894, 92477, 56089.
11. By 12: 348, 564, 936, 3888, 5737, 20928, 3708, 94020, 67308.

47. *Ex.*—How often is 6 contained in 24295?

Dividing as before, there is a remainder of one after dividing the units. This is annexed to the quotient with the divisor below in the form $\frac{1}{6}$, which denotes *one-sixth*, or the sixth part of one.

$$6\overline{)24295}\quad 4049\tfrac{1}{6}$$

In multiplying the quotient in this case by the divisor to prove the result, the remainder must be added to the product; thus, $4049 \times 6 + 1 = 24295$.

EXERCISE VII.

Divide—	1.	2.	3.	4.	5.	6.
1. By 2,	345	467	931	857	1129	2525
2. By 3,	472	305	721	922	2684	7055
3. By 4,	105	653	437	829	5634	8631
4. By 5,	732	482	911	573	8421	7018
5. By 6,	515	833	791	273	5927	6381
6. By 7,	452	635	134	608	3210	7962
7. By 8,	123	537	817	909	4561	8347
8. By 9,	258	316	501	823	7082	1293
9. By 10,	137	259	533	471	2563	9327
10. By 11,	564	800	601	942	3874	6088
11. By 12,	373	529	705	637	1949	2009

48. Division by Factors.

In dividing by any composite number up to 144, we may get the quotient by dividing by its two factors successively. *E.g.*, in dividing an apple into 4 parts, we first divide it into 2 parts, then each of these again into 2 parts.

Ex.—Divide 3568 marbles into parcels of 24.

The factors of 24 are 6 and 4.

Dividing first by 6, we have for quotient 594 (parcels of 6), and 4 (marbles) over.

Dividing next by 4, we have for quotient 148 (parcels of 4 sixes or 24's) and 2 (parcels of 6) over.

Adding now the second remainder (2 parcels of 6, or 12 marbles) to the first (4 marbles), we have for total remainder 16 marbles: $6 \times 2 + 4 = 16$.

$$\begin{array}{r|l} 6 & 3568 \\ \hline 4 & 594 - 4 \\ \hline & 148 - 2 \end{array} \Big\} 16$$

Hence, to get the real remainder, multiply the first divisor by the second remainder, and add the first remainder to the product. If there be no second remainder, the first is the real one.

EXERCISE VIII.

1. $23456 \div 14$, 15, 21, 22
2. $37095 \div 25$, 27, 28, 32
3. $90851 \div 33$, 42, 44, 45
4. $84379 \div 54$, 55, 56, 63
5. $65927 \div 66$, 77, 81
6. $905036 \div 84$, 88, 96
7. $249076 \div 99$, 108
8. $593250 \div 120$, 132, 144
9. $731105 \div 16$, 18, 48, 72, in two ways.
10. $847644 \div 24$, 36, in three ways.

49. Division by more than one Place.

As a cipher annexed to the right of a figure multiplies it by 10, so a cipher removed from the right of a figure divides the number by 10: thus, $20 \div 10 = 2$.

If the dividend do not end in a cipher, then the figure in the units' place is removed for a remainder: thus, $21 \div 10 = 2\frac{1}{10}$.

If the divisor contain more tens than one, as 30, divide first by 10 as one factor, and then by the other factor, 3; that is, remove the units' place of the dividend for the remainder, and divide by the second factor, carrying what is over in this division to the remainder. Thus, $63 \div 20 = 3\frac{3}{20}$; $73 \div 20 = 3\frac{13}{20}$.

To divide by a number of hundreds, remove the two last ciphers of the dividend, or the two last figures of it, for remainder, in a similar way. Thus, $200 \div 100 = 2$; $564 \div 200 = 2\frac{164}{200}$.

EXERCISE IX.

Divide by 10, 30, 50, 70, 90—

1. 370	7. 1200	13. 2474	19. 32814
2. 290	8. 6600	14. 3935	20. 56732
3. 835	9. 8800	15. 5066	21. 83940
4. 672	10. 7000	16. 7317	22. 50761
5. 425	11. 4800	17. 8058	23. 69005
6. 901	12. 6300	18. 9720	24. 85436

EXERCISE X.

Divide by 200, 400, 600, 800, examples 7-24 in last Exercise.

50. *Ex.*—How often is 234 contained in 849726?

234 in 8 or in 84 cannot be taken, but in 849 (thousands) is 3 (thousands), and 147 (thousands) over. Set down the 3 in the thousands' place of the quotient, and carry the 147 to the hundreds' place, making the next part of the dividend 1477 (hundreds) in all.

234 in 1477 (hunds.) is 6 (hundred), and 73 (hunds.) over. Set down the 6 (hunds.) in its place in the quotient, and carry the 73 (hunds.) to the tens' place, making the next part of the dividend 732 (tens) in all.

234 in 732 (tens) is 3 (tens), and 30 (tens) over. Set down the 3 (tens) in its place in the quotient, and carry the 30 (tens) to the units' place, making the next part of the dividend 306 (units) in all.

234 in 306 (units) is 1 (unit), and 72 (units) over. Set the 1 (unit) in its place in the quotient. The 72 units are remainder.

```
234)849726(3631 72/234
  702
  ---
 1477
 1404
 ----
  732
  702
  ---
  306
  234
  ---
   72
```

This form of division, which is required when the divisor contains more than one place, is known as *Long Division.*

EXERCISE XI.

1. 37037÷25, 37, 43.
2. 29835÷34, 49, 51.
3. 73632÷47, 93, 39.
4. 80294÷19, 26, 41.
5. 90000÷73, 61, 17.
6. 50032÷29, 53, 98.
7. 17918÷13, 34, 82.
8. 47320÷38, 91, 47.
9. 20971÷67, 82, 93.
10. 54280÷23, 46, 85.

EXERCISE XII.

1. 45682÷251, 183, 342.
2. 40936÷301, 457, 631.
3. 23843÷113, 911, 564.
4. 89040÷824, 159, 296.
5. 90000÷457, 734, 825.
6. 12384÷391, 516, 364.
7. 73027÷801, 709, 208.
8. 29041÷257, 314, 846.
9. 92881÷934, 652, 293.
10. 79948÷418, 506, 853.
11. 560802÷293, 791, 846.
12. 293544÷151, 258, 174.
13. 858841÷325, 291, 397.
14. 485361÷851, 702, 813.
15. 934110÷561, 582, 738.
16. 500800÷921, 309, 257.
17. 700000÷416, 526, 736.
18. 205806÷901, 754, 815.
19. 934165÷297, 358, 492.
20. 714408÷824, 964, 708.

EXERCISE XIII.

1. 7489318÷37, 74, 89.
2. 2934821÷41, 73, 97.
3. 7348640÷594, 416, 607.
4. 3584816÷208, 541, 732.
5. 6084516÷2342, 5684.
6. 5403144÷9348, 2571.
7. 7256154÷3040, 8009.
8. 9144668÷9401, 5008.
9. 8271759÷3075, 4908.
10. 9193932÷5671, 2943.
11. 57338064÷5437, 3024, 9902.
12. 63092706÷2931, 4708, 5004.
13. 72491840÷3040, 8009, 5231.
14. 20018414÷7298, 6804, 7734.
15. 92100625÷5136, 1984, 2875.
16. 80000000÷8345, 6205, 7095.
17. 53805448÷4001, 8936, 9027.
18. 7300692÷7506, 9324.
19. 9000000÷8931, 7295.
20. 8203570÷4583, 9308.
21. 256890368÷28, 79, 39.
22. 931456204÷17, 47, 82.
23. 249086022÷457, 329, 704.
24. 303606796÷293, 718, 274.
25. 724088043÷8561, 2793.
26. 365905780÷5006, 2918.
27. 854372400÷9300, 8540.
28. 293600170÷2005, 7009.
29. 875912780÷3054, 4090.
30. 293400000÷7200, 5090.

51. To find an Average.

Ex.—A boy gets 23 marks on Monday, 17 on Tuesday, 28 on Wednesday, 31 on Thursday, 25 on Friday, and 14 on Saturday: what is his average number of marks daily for the week?

Here the sum of his marks for the whole week is 138. There is a certain number of marks, which had he got every day of the week, the sum of his marks at the end of the week would

have been the same as it is now. That is the number we wish to find.

The *average* of a series of numbers is that number which, if repeated as often as there are numbers, will amount to their sum. It is found by dividing the *sum* of the numbers by their *number;* thus $138 \div 6 = 23$.

EXERCISE XIV.

Find the average of the following numbers :—

1. 27, 37, 42, 50, 22, 24.
2. 13, 49, 35, 64, 53, 42.
3. 93, 87, 59, 67, 73.
4. 29, 30, 37, 32, 33.
5. 125, 250, 315, 193.
6. 2738, 3624, 3001,
7. 937, 1001, 1100, 1010, 1110.
8. 856, 1533, 930, 1399.
9. 8973, 10704, 9320, 14976, 9999.
10. 27345, 73421, 85648, 79286.

52. Fractional Multipliers and Divisors.

Ex.—A train runs 27 miles an hour for $14\frac{3}{4}$ hours; what distance will it go in the time?

The distance is 27 miles repeated 14 times and $\frac{3}{4}$ a time; which is got by multiplying 27 by $14\frac{3}{4}$.

To multiply by $\frac{3}{4}$, multiply by 3 and divide the product by 4. Then in multiplying by 14, the right-hand figure of the first line, being units, is set in the units' place.

$$\begin{array}{r} 27 \\ 14\frac{3}{4} \\ \hline 4)81 \\ \hline 20\frac{1}{4} \\ 108 \\ 27 \\ \hline 398\frac{1}{4} \end{array}$$

The number $\frac{3}{4}$, which is less than 1 is called a *fraction.*

If one is divided into 2 equal parts, each is called *a half;* if into 3, each is called a third; if into 4, a fourth; and so on. A fraction is denoted by two numbers, the one written below the other; thus one-half is written $\frac{1}{2}$, one-third $\frac{1}{3}$, one-fourth $\frac{1}{4}$; if more than one part be taken, the upper figure denotes how many, thus three-fourths is written $\frac{3}{4}$. The number $14\frac{3}{4}$, which consists of a whole number and a fraction, is called a *mixed number.*

EXERCISE XV.

1. Find one-half of 38, 57, 108, 265, 798, 6357.
2. One-third of 51, 252, 254, 768, 784, 8472.
3. One-fourth of 56, 92, 94, 397, 3828, 8927.
4. Multiply by $\frac{3}{4}$: 85, 101, 357, 456, 2456, 7530.
5. Multiply by $\frac{2}{3}$: 84, 356, 537, 933, 1272, 7000.
6. $3456 \times 4\frac{1}{4}$, $6\frac{3}{4}$, $15\frac{1}{4}$, $27\frac{3}{4}$, $139\frac{3}{4}$, $308\frac{1}{4}$.
7. $93582 \times 10\frac{3}{4}$, $200\frac{1}{4}$, $750\frac{2}{3}$, $409\frac{1}{2}$, $30\frac{1}{4}$, $5\frac{1}{2}$.

Ex.—How often is $29\frac{1}{4}$ contained in 9384?

The numbers cannot conveniently be used for divisor and dividend as they stand.

Multiply both by 4, the fraction in the divisor being three-*fourths*. This will give a new divisor and dividend four times greater than those given; but which will be free from fractions, and will give the same quotient.

$29\frac{1}{4}$ 9384
4 4
117)37536(320$^{96}_{117}$
351
243
234
96

EXERCISE XVI.

1. $3482 \div 3\frac{1}{2}$, $6\frac{2}{3}$, $8\frac{1}{4}$.
2. $8506 \div 4\frac{1}{4}$, $5\frac{1}{2}$, $9\frac{1}{3}$.
3. $72584 \div 27\frac{1}{2}$, $54\frac{2}{3}$, $79\frac{3}{4}$.
4. $59321 \div 19\frac{1}{3}$, $68\frac{3}{4}$, $128\frac{1}{4}$.
5. $80999 \div 15\frac{3}{7}$, $26\frac{3}{5}$, $94\frac{3}{8}$.
6. $900536 \div 12\frac{4}{5}$, $74\frac{1}{3}$, $256\frac{1}{2}$.
7. $852079 \div 5\frac{1}{2}$, $30\frac{1}{4}$, $365\frac{1}{4}$.
8. $205930 \div 15\frac{3}{4}$, $85\frac{1}{4}$, $365\frac{1}{4}$.
9. $7305267 \div 29\frac{1}{3}$, $217\frac{3}{5}$, $8342\frac{1}{16}$.
10. $45067824 \div 14\frac{3}{4}$, $58\frac{1}{2}$, $100\frac{1}{2}$.

53. Multiplication and Division Combined.

Ex.—What number results from multiplying 57 by 16, and dividing by 24?

To multiply by 16 is the same as to multiply by 2 and then by 8; and to divide by 24 is the same as to divide by 3 and and then by 8. We may strike out the 8 from both terms; since to multiply a number by 8 and then to divide it by 8 leaves it unaltered. So that—

$$\frac{57 \times 16}{24} = \frac{57 \times \cancel{16}^{2}}{\cancel{24}_{3}} = 38.$$

The striking out of a factor common to a multiplier and a divisor is called *cancelling*. Cancelling may sometimes be performed more than once in the same exercise; thus—

$$\frac{\cancel{6}^{1} \quad \cancel{9}^{1}}{\cancel{24}_{4} \times \cancel{9}_{1}} = 6.$$

EXERCISE XVII.

Perform the following operations, cancelling where possible.

1. $\frac{9 \times 7}{3}$ $\frac{8 \times 15}{5}$ $\frac{24 \times 12}{18}$ $\frac{16 \times 6}{8}$ $\frac{33 \times 14}{35}$ $\frac{48 \times 24.}{72}$
2. $\frac{45 \times 36}{81}$ $\frac{84 \times 48}{84}$ $\frac{105 \times 21}{49}$ $\frac{117 \times 48}{108}$ $\frac{57 \times 25}{40}$ $\frac{38 \times 28}{44}$
3. $\frac{89 \times 32}{72}$ $\frac{157 \times 81}{108}$ $\frac{238 \times 63}{119}$ $\frac{181 \times 36}{54}$ $\frac{66 \times 45}{99}$ $\frac{124 \times 18}{60}$

4. $\frac{35\times9\times12}{4\times18}$ $\frac{45\times16\times18}{36\times45}$ $\frac{24\times15\times21}{40\times35}$ $\frac{30\times14\times24}{20\times28}$ $\frac{42\times16\times32}{48\times35}$

5. $\frac{59\times10\times33}{11\times60}$ $\frac{63\times8\times25}{35\times32}$ $\frac{18\times14\times28}{9\times36}$ $\frac{50\times34\times21}{14\times25}$ $\frac{9\times8\times6}{3\times4}$

6. $\frac{147\times24\times18}{72\times45}$ $\frac{240\times65\times8}{16\times30}$ $\frac{306\times28\times63}{35\times102}$ $\frac{564\times84\times33}{88\times144}$

Any number is divisible exactly—

1. By 2, when its last place is divisible by 2.
2. By 4, when its last two places are divisible by 4.
3. By 8, when its last three places are divisible by 8.
4. By 3,
5. By 9, } when the sum of its places is divisible by 3 or 9.
6. By 5, when its last place is 5 or 0.
7. By 10, when its last place is 0.

EXERCISE XVIII.

1. $\frac{243\times316}{228}$ $\frac{79\times104}{432}$ $\frac{348\times252}{384}$ $\frac{219\times573}{693}$ $\frac{391\times215}{300}$ $\frac{893\times4128}{376}$

2. $\frac{256\times216}{852}$ $\frac{750\times375}{265}$ $\frac{358\times516}{372}$ $\frac{250\times700}{8000}$ $\frac{295\times415}{375}$ $\frac{312\times462}{294}$

3. $\frac{584\times2928}{3024}$ $\frac{73\times321}{412}$ $\frac{92\times840}{342}$ $\frac{300\times200}{6000}$ $\frac{843\times356\times296}{296\times560}$

54.

EXERCISE XIX.

1. How many scores in 340?
2. How many one-dozen baskets may be filled out of 468 bottles?
3. How many pieces, each 25 yards, may be got from 6425 yards.
4. How many forms, of 15 each, will hold 675 scholars?
5. Into how many parcels of 16 may 432 marbles be divided?
6. How often can I subtract 64 from 2304?
7. What must 73 be multiplied by to give 22995?
8. How many regiments, each 829, are in an army of 38963 men?
9. If 2664 be dividend, and 36 be quotient, find the divisor.
10. How many boxes will hold 7000 oranges, if each hold 125?
11. If a man divides £728 equally among his 4 children, what is the share of each?
12. How many years' rent of a house at £6 is £792?
13. If the journey from London to Edinburgh, which is 385 miles, be made in 11 hours, what rate is that per hour?
14. What multiplier of 346 gives 81964 as product?
15. If a tradesman saves 5 shillings a week, in how many weeks will he save 850 shillings.
16. What is the nearest number to 850 which can be divided evenly by 27? and the next nearest?
17. The year 1864 began on a Friday, how many Fridays had it? and how many Sundays?

18. In a certain city there died in the month of April 23790 persons, what was the daily number of deaths on an average?

19. A banker has a box with 7460 shillings, 24 five-shilling pieces, and 50 florins, how often can he change a pound?

20. Five trains left London Bridge for the Crystal Palace, the first with 379 passengers, the second with 250, the third with 483, the fourth with 579, and the fifth with 294: what was the average number in each train?

21. A regiment of 1170 men had one man killed or wounded in battle for every 18 men in it: how many remained fit for service?

22. A cargo of tea, 435 chests, each 180 pounds' weight, is to be packed in boxes, each containing 54 pounds: how many of these must be ordered?

23. What must I add to the square of 154 to contain exactly the square of 27?

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55. MISCELLANEOUS EXERCISE ON THE FOUR RULES.—I.

1. Printing was invented 1440 A.D., and the first book was printed in England 34 years thereafter: what was its date?

2. If a farmer sells 35 oxen for £12 each, 253 sheep for £2 each, and 159 lambs at £1 each, what does he receive for all?

3. The circumference of the earth is 24900 miles, in how many days could a ship sail round it at 9½ miles an hour?

4. How much higher is Mont Blanc, the highest mountain in Europe, which is 15,680 feet high, than Ben Nevis, the highest in Britain, which is 4368 feet high?

5. To half the sum of 85 and 57 add half their difference.

6. A clerk, engaged for five years, receives £80 salary the first year, and an advance of £15 each year: what is his average yearly salary?

7. The six largest cities in England are London with 2,362,236 inhabitants, Liverpool with 375,955, Manchester with 316,213, Birmingham with 232,841, Leeds with 172,000, and Bristol with 137,000: what is the population of these cities together?

8. Sir Isaac Newton was born in 1642 and died in 1729: how old was he at his death?

9. Three apples were given to each of 178 pupils of a school, but 672 apples were provided in all: how many more pupils could have been served?

10. I met 7 flocks of sheep, of one score each, on their way to market, 5 of twoscore and nine each, 6 of threescore and ten each, and then one of 19: how many sheep did I pass?

11. From London to Peterborough is 76 miles, from Peterborough to York 115 miles, from York to Newcastle 72 miles, from Newcastle to Berwick 65 miles, from Berwick to Edinburgh 57 miles: what is the distance from London to Edinburgh?

12. What number added to 7803 will make up the third part of 87003?

13. To 7 times the sum of 909 and 98, add 7 times their difference.

14. A train contains 1097 passengers; of these, 286 are first-class, and half as many more second-class: how many third-class are there?

15. What divisor of 44934 gives 348 as quotient, and 42 over?

16. Find the number of days in a leap year.

17. A teacher buys 100 boxes steel-pens, containing one gross each. He has 563 pupils in school: after serving them with pens 7 times, how many remain?

18. The ship "Graceful," from Charente to Leith, discharged 2552 one-dozen cases brandy, 122 two-dozen cases, and 16 three-dozen cases: how many gross of bottles were in her cargo? If 6 bottles go to a gallon, how many gallons of brandy?

19. A shelf in a library contained—History of England, 10 volumes; British Poets, 75 volumes; Goldsmith's Works, 4 volumes; Waverley Novels, 25 volumes; British Essayists, 45 volumes; and the shelf below contained exactly the same number: how many volumes were on both?

20. What must be added to the third part of 1395 to bring it up to the fifth part of 3790?

21. Find the product of three numbers, of which the first, 374, exceeds the second by 93, and the third by twice as much.

22. In what time will 3 pipes empty a tank of 429165 gallons, if they run off respectively 450, 500, and 535 gallons per hour?

23. If a stage-coach travel $5\frac{1}{2}$ miles an hour, how far will it go in two days of 9 hours each?

24. An army of 69776 men was drawn up in squares of 28 in a side; how many squares were there?

25. Find the difference between the square of 9009, and the cube of 909.

MISCELLANEOUS EXERCISES—*continued.* II.

1. Julius Cæsar invaded Britain 55 B.C.: how long was that before the union of England and Scotland in 1700?

2. How often does a clock strike in a year?

3. A boy, working 8 hours a day, can point in a year 33979280 pins: how many can he point in an hour?

4. A travels 3 miles an hour, B $4\frac{1}{2}$: when B has gone 45 miles, how far has A gone?

5. Great Britain and Ireland contain 121385 square miles; the British possessions in Europe, 145; in Asia, 928610; in North America, 768577; in South America, 89000; in Africa, 201403; in the West Indies, 73384; in Australasia, 560000. What is the whole area of the British Empire?

6. Michaelmas is 86 clear days before Christmas: what is the date of it?

7. January 4, paid into savings'-bank, 14 shillings; February 1, paid in 13 shillings; February 28, drew out 11 shillings; March 14, paid in 19 shillings; March 31, drew out 25 shillings; April 24, paid in 17 shillings; May 3, paid in 9 shillings; May 25, drew out 15 shillings; June 1, paid in 16 shillings. My account was then balanced: how much had I at my credit?

8. Adam lived 930 years; Seth, his son, was born when he was 130 years old, and lived 912 years: how long did they live together?

9. A bag of nuts, containing 3000, was divided among a school; the pupils above 9 years got 35 each, and those below 9 (who were exactly the same number) got 25 each: how many pupils were in the school?

10. A railway guard makes two journeys every lawful day from

Edinburgh to Glasgow and back; if these towns are 47 miles apart, what distance has he travelled, after being in his situation five years?

11. Three regiments form squares, the side of the first being 33 men, of the second 29, and of the third 27: how much stronger is the first regiment than the second, and the second than the third?

12. How often will a cart-wheel, $16\frac{1}{2}$ feet round, revolve in going a mile, which has 5280 feet?

13. A railway 273 miles long has a station every $10\frac{1}{2}$ miles on the average: how many stations has it? And what is the length of a railway which has 18 stations, distant on the average $7\frac{1}{2}$ miles from each other?

14. George I. of England began to reign 1714 A.D., and reigned 13 years; George II. reigned 33 years, George III. 60, George IV. 10, and William IV. 7 years. Queen Victoria succeeded William; in what year did she begin to reign?

15. The sea route from London to Hamburgh is 482 miles. When the London steamer is 130 miles on its way, and the Hamburgh steamer 210 miles on its, how far are they apart?

16. If Scotland produced in 1864, 23000 tons pig-iron weekly, what was the produce for the year? and at £3 a ton, how much did it add to the wealth of the country during the year?

17. A farm has 5 fields, the first containing 89 acres, the second 101, the third 174, the fourth 92, and the fifth the average of the other four. It is to be divided into as many fields of equal size: how many acres will each contain?

18. (*a*) A legacy of £1595 is left to two charities, of which the one receives half as much again as the other: what was the share of each?

(*b*) Out of a legacy of £8570, £730 were devoted to charitable purposes; the rest was to be divided into 9 shares, of which the eldest son was to get four, the second three, and the youngest two: how much did each get?

19. If a candidate at an election is returned by a majority of 291 votes out of 3579, how many voted for the unsuccessful candidate?

20. If I bought 79 shares in the Great Western Railway at £64 each, and sold out at £69, what did I pay for them, and what did I gain?

21. The exports from Liverpool to the United States in 1861 were £8223587; in 1862, £11986233; and in 1863, £13765217. How much did the increase in 1862 exceed that in 1863?

22. In a journey of 37 hours, I travelled one-third of the time at 24 miles an hour, and two-thirds at 27 miles: what was the length of the journey?

23. Divide 318 apples among 18 boys and 8 girls, giving each boy twice and a half as much as a girl.

24. Handel, the great musical composer, died in 1759, aged 75; and Haydn was born when Handel was in his seventeenth year; what year was that?

25. A tank of water contained 75000 gallons. A supply was drawn off by 3 pipes, which ran for 10 hours at the rate of 255 gallons each per hour; but during that time two pipes ran into the tank 335 gallons each per hour: how much water was left?

26. Find the difference between the square of the sum of 28 and 39, and the sum of their squares.

27. A sum of money was divided between A, B, C, and D, so that A got £260, B £375, C the excess of B's share above A's, whilst D was to receive £25 from A, £48 from B, and £17 from C, as his share. What were the shares of all four?

28. The sum of 2 numbers is 428, and their difference 194; find the numbers.

*** Half the sum + half the difference gives the greater.
Half the sum − half the difference gives the less.

29. A tradesman, out of his weekly savings for a year, bought a table that cost 22s., 6 chairs that cost 7s. each, a carpet of 20 square yards in size at 3s. a yard; he had besides 32s. over: how much had he saved every week?

30. A farmer paid £780 for cows and sheep. Of this sum he paid £350 for 25 cows; if a cow cost 7 times as much as a sheep, how many sheep did he buy with the rest of his money?

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56. MONEY.

MONEY OF ACCOUNT—TABLE I.

Accounts are kept in pounds, shillings, pence, and farthings sterling.

Pounds are denoted by the letter £, thus £40.

Shillings by the letter s., or by a line; thus, 3s. or 3/.

Pence by the letter *d.*, thus 9d.[1]

Farthings, which mean *fourths of a penny*, are denoted by fractional numbers; thus, one farthing by $\frac{1}{4}$; two farthings, or one halfpenny, by $\frac{2}{4}$d.; and three farthings by $\frac{3}{4}$d.

Pounds, shillings, and pence, when written in columns, are denoted by £ *s. d.* placed over the column.

EXERCISE.

Read off the sums in Ex. i. sect. 58.

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57. COMPOUND ADDITION.

Ex.—If I have paid into the bank in January, £27, 11s. $3\frac{1}{4}$d.; in February, £23, 14s. $8\frac{1}{2}$d.; in March, £13, 19s. $9\frac{3}{4}$d.; in April, £7, 0s. $2\frac{1}{4}$d.; in May, £2, 7s. 11d.; and in June, 17s. $3\frac{1}{2}$d.: how much have I put in during the six months?

We have here to find the sum of the six payments; which we do by addition.

Write the numbers below each other, pounds below pounds, shillings below shillings, and pence below pence.

[1] £ is the first letter of *libra*, a Roman weight; *s.* and *d.* the first letters of *solidus* and *denarius*, Roman coins.

Then, adding the farthings' column, we have 2-3-6-8-9 farthings, which is $2\frac{1}{4}$d. ; set down the $\frac{1}{4}$d. and carry the 2d.

Adding the pence column, we have, by simple addition, 29d., that is 2/5 ; set down the 5d. and carry the 2/.

Adding the shillings' column, we have, by simple addition, 70/, that is £3, 10s. ; set down the 10s., and carry the £3.

Adding the pounds' column, we have, by simple addition, £75.

Sum, £75, 10s. $5\frac{1}{4}$d.

£	s.	d.
27	11	$3\frac{1}{4}$
23	14	$8\frac{3}{4}$
13	19	$0\frac{3}{4}$
7	0	$2\frac{1}{4}$
2	7	11
0	17	$3\frac{1}{2}$
£75	10	$5\frac{1}{4}$

Rule.—Write the numbers below each other so that each column may be of the same name ; add each column in its order, carrying as many of the next highest name as are contained in its sum.

The result may be proved, as in simple addition, by adding the columns from the top downward.

The addition of *quantities of different names*, as here, is called *compound addition*.

EXERCISE I.

58.

	1.			2.			3.			4.			5.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
a.	3	6	$10\frac{1}{2}$	2	10	$9\frac{1}{4}$	11	9	$10\frac{3}{4}$	7	9	$4\frac{1}{2}$	5	10	$9\frac{1}{2}$
b.	4	16	$8\frac{1}{4}$	3	8	$4\frac{1}{2}$	5	4	$3\frac{1}{2}$	11	9	$2\frac{1}{4}$	7	9	$6\frac{1}{4}$
c.	9	3	$0\frac{1}{2}$	9	5	$6\frac{3}{4}$	7	2	$9\frac{1}{4}$	5	0	$7\frac{1}{4}$	1	14	$10\frac{1}{2}$
d.	5	14	$10\frac{1}{4}$	7	7	$10\frac{1}{2}$	8	0	$0\frac{3}{4}$	3	17	$2\frac{1}{2}$	3	9	$6\frac{1}{2}$
e.	10	0	9	4	15	11	7	2	$11\frac{1}{4}$	11	9	$5\frac{1}{4}$	0	17	10
f.	6	18	$6\frac{1}{2}$	7	2	9	0	8	$2\frac{1}{4}$	8	2	$4\frac{1}{4}$	2	17	$9\frac{3}{4}$
g.	0	19	$3\frac{1}{4}$	12	0	$5\frac{3}{4}$	9	0	11	12	0	$0\frac{3}{4}$	8	5	$10\frac{1}{2}$
h.	11	5	$8\frac{1}{2}$	6	16	$10\frac{1}{2}$	7	10	$9\frac{1}{2}$	9	7	6	11	11	$4\frac{3}{4}$
i.	1	1	$1\frac{1}{4}$	8	2	$3\frac{1}{2}$	11	15	0	0	18	$9\frac{3}{4}$	8	7	$9\frac{3}{4}$
j.	2	3	$9\frac{3}{4}$	7	5	$6\frac{1}{4}$	4	10	11	10	9	$4\frac{1}{2}$	3	11	$5\frac{1}{4}$
k.	12	2	10	10	11	$11\frac{1}{4}$	12	12	$9\frac{1}{2}$	5	0	0	2	7	6
l.	9	6	$5\frac{1}{4}$	2	19	6	1	17	$1\frac{3}{4}$	7	0	$10\frac{1}{2}$	9	9	$4\frac{1}{4}$
m.	8	10	$11\frac{1}{2}$	3	15	4	2	4	$8\frac{1}{2}$	5	4	$9\frac{1}{4}$	12	4	$10\frac{1}{2}$
n.	5	14	$3\frac{1}{4}$	4	17	$2\frac{1}{2}$	5	15	$7\frac{1}{2}$	6	10	$10\frac{1}{2}$	1	19	$11\frac{1}{4}$
o.	0	10	2	5	10	$11\frac{3}{4}$	10	0	0	0	18	$2\frac{3}{4}$	6	9	$6\frac{1}{2}$
p.	1	17	$9\frac{1}{4}$	11	9	4	9	0	9	11	9	$2\frac{1}{4}$	11	4	$4\frac{1}{2}$

EXERCISE II.

1. Count from 1d., 2d., 3d., etc., by $1\frac{1}{2}$d., $2\frac{1}{2}$d., $2\frac{3}{4}$d., $4\frac{1}{4}$d., etc.
2. Count from 1 sh., 2 sh., 3 sh., etc., by 1/3, 2/4, $1/3\frac{1}{2}$, etc.
3. Count from £3, £4, £5, etc., by 7/8, 13/4, $12/6\frac{1}{2}$, etc.

*** Ex. i. and ii. for oral practice, whilst the pupil is working the following Exercise. The same remark applies to the subsequent rules.

59.

EXERCISE III.

1.			2.			3.			4.			5.		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
73	10	$9\frac{1}{2}$	14	10	$8\frac{1}{4}$	47	17	10	63	14	$5\frac{3}{4}$	72	8	$4\frac{1}{4}$
47	5	$0\frac{3}{4}$	92	0	$5\frac{1}{2}$	39	14	$8\frac{3}{4}$	28	10	10	17	8	$9\frac{3}{4}$
3	7	$9\frac{1}{4}$	37	16	4	7	19	$10\frac{1}{4}$	37	15	$7\frac{1}{4}$	93	15	$2\frac{1}{4}$
13	17	4	29	18	$11\frac{3}{4}$	72	12	$6\frac{1}{2}$	9	9	$9\frac{1}{4}$	82	10	$4\frac{3}{4}$
28	9	$3\frac{3}{4}$	15	14	$0\frac{1}{4}$	84	0	$11\frac{1}{4}$	8	5	$7\frac{3}{4}$	79	1	11
80	5	11	34	8	5	59	10	$0\frac{3}{4}$	92	10	$10\frac{1}{2}$	9	11	$1\frac{1}{4}$

6.			7.			8.			9.			10.		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
42	8	$8\frac{1}{4}$	147	2	$3\frac{1}{2}$	293	14	$10\frac{1}{4}$	673	10	$8\frac{1}{4}$	534	10	9
28	10	$0\frac{3}{4}$	82	5	$11\frac{1}{4}$	118	10	$0\frac{1}{4}$	200	18	0	27	0	6
59	0	$0\frac{1}{4}$	7	17	10	500	8	$0\frac{3}{4}$	74	0	11	8	5	$0\frac{3}{4}$
72	0	$10\frac{1}{2}$	973	0	$7\frac{1}{4}$	94	3	$6\frac{3}{4}$	9	10	$4\frac{3}{4}$	904	15	2
5	18	1	459	19	$11\frac{3}{4}$	7	18	2	28	14	$8\frac{1}{4}$	673	17	$0\frac{1}{4}$
38	14	11	226	4	$0\frac{1}{2}$	192	17	$5\frac{1}{4}$	990	19	$0\frac{1}{2}$	49	15	$8\frac{1}{2}$
0	19	$3\frac{1}{4}$	305	2	11	201	0	$7\frac{1}{2}$	309	17	6	200	18	2
17	8	$6\frac{1}{4}$	38	18	$6\frac{1}{2}$	802	14	0	25	8	0	55	16	0

11. £934, 18, 6+£84, 0, 9+£702, 15, $2\frac{1}{4}$+£39, 4, 0+£740, 0, 0+£85, 16, $2\frac{1}{4}$+£156, 18, $6\frac{1}{4}$+£529, 5, $1\frac{1}{2}$.

12. £617, 10, $11\frac{1}{4}$+£290, 0, $10\frac{1}{2}$+£38, 5, 6+£93, 0, $0\frac{1}{4}$+£549, 7, $2\frac{1}{4}$+£29, 10, 0+£709, 18, $4\frac{3}{4}$+£815, 16, 1.

13. £127, 14, $8\frac{1}{4}$+£293, 11, $5\frac{1}{2}$+£340, 10, $10\frac{1}{4}$+£458, 10, 9+£500, 17, $7\frac{3}{4}$+£110, 19, $2\frac{1}{4}$+£301, 1, $11\frac{1}{4}$+£824, 0, $0\frac{1}{2}$+£629, 5, 5.

14. £543, 10, $0\frac{1}{4}$+£94, 17+£7, 10, $6\frac{3}{4}$+£829, 7+£471, 10, $9\frac{1}{4}$+£28, 15, $0\frac{1}{2}$+£728, 16, $10\frac{1}{4}$+£840, 0. 11.

15. £293, 18+£72, 19, $1\frac{1}{4}$+£9, 10, $5\frac{3}{4}$+£820, 15+£94, 18, $6\frac{3}{4}$+£571, 15, $4\frac{1}{4}$+£629, 18, 4+£930, 15, $10\frac{1}{2}$.

16. £2005, 7, 6+£943, 18, $1\frac{1}{4}$+£564, 9+£7248, 0, $9\frac{1}{2}$+£1508, 10, $8\frac{3}{4}$+£592, 8, $0\frac{1}{4}$+£9408, 2, 10+£93, 0, $11\frac{1}{4}$.

17. £329, 14, $4\frac{1}{2}$+£73, 18, $5\frac{1}{2}$+£493, 9, $4\frac{1}{2}$+£701, 1, $7\frac{1}{2}$+£592, 10, 11+£17, 4, 8+£9, 7, $6\frac{1}{4}$+£341, 19, $8\frac{3}{4}$+£700, 1, 11.

18. £112, 9, $4\frac{1}{2}$+£257, 3, $0\frac{1}{4}$+£562, 11, $7\frac{3}{4}$+£79, 19, $9\frac{1}{4}$+£790, 8, 2+173, 13, $11\frac{3}{4}$+£459, 12, 10+£614, 14, $11\frac{1}{2}$+998, 19, $5\frac{1}{4}$.

19. £72, 7, 10+£394, 6, $4\frac{1}{4}$+£593, 0, $8\frac{1}{4}$+£360, 0, $11\frac{3}{4}$+£94, 15, $8\frac{1}{4}$+£250, 11+£37, 18, $0\frac{1}{4}$+£84, 15, $6\frac{3}{4}$+£420, 18, 6+£13, 2, $1\frac{1}{4}$.

20. £640, 10, 11+£93, 4, $7\frac{1}{2}$+£870, 19+£250, 0, $9\frac{3}{4}$+£550, 9, 1+£709, 13, $6\frac{3}{4}$+£1, 2, $3\frac{1}{4}$+£85, 16, 6+£924, 15, $1\frac{1}{2}$+£9, 2, $8\frac{3}{4}$.

21. £279, 18, 6+£90, 17, $3\frac{1}{4}$+£250, 4, 10+£79, 18, $1\frac{3}{4}$+£100, 15+£25, 0, $6\frac{1}{4}$+£365, 19, 1+£209, 14, $7\frac{1}{4}$+£99, 18, 4+£805, 7, $6\frac{1}{4}$.

22. £8408, 14, 10+£2930, 10, $4\frac{1}{4}$+£6009, 19, $0\frac{3}{4}$+£509, 7, $11\frac{1}{4}$+£93, 10, $6\frac{1}{4}$+£793, 10, $0\frac{1}{4}$+£209, 18, 1+£3085, 2, $0\frac{3}{4}$+£94, 18, $2\frac{1}{4}$.

23. £2563, 14, 1+£846, 10, $0\frac{1}{4}$+£2564, 0, $10\frac{1}{2}$+£865, 17, $11\frac{3}{4}$+£590, 0, 6+£859, 2, $1\frac{1}{4}$+£9337, 19, $0\frac{1}{2}$+£820, 7, 6+£94, 17, $6\frac{3}{4}$.

EXERCISE IV.

Work the questions Ex. iii. as directed Ex. xi. p. 15.

60. COMPOUND SUBTRACTION.

Ex.—If I pay a debt of £28, 18s. 5½d. out of a sum of £63, 13s. 4¼d., how much have I over?

We have here to find the difference of these two sums of money; which we do by subtraction.

Write the subtrahend below the minuend in its place.

2 f. from 1 f. cannot be taken; change one of the pence, making 5 f. in all; 2 f. from 5 f. leaves 3 f.

5d. from 3d. cannot be taken; change one of the shillings, making 15d. in all; 5d. from 15d. leaves 10d.

18s from 12s. cannot be taken; change one of the pounds, making 32s. in all; 18s. from 32s. leaves 14s.

28 pounds from 62 pounds leaves 34.

£	s.	d.
63	13	4¼
28	18	5½
£34	14	10¾

Rule.—Write the subtrahend below the minuend so that each column shall be of the same name; subtract each column in its order, changing one of the next highest name when necessary.

The result may be proved, as in simple subtraction, by adding together the subtrahend and the difference.

The subtraction of *quantities of different names*, as here, is called *compound subtraction.*

Or thus:[1]

Then, beginning with the lowest name, 2 from 1 cannot be taken; add 1d. or 4 farthings, making 5f. in the minuend; 2f. from 5f. is 3f.

Then 6d. from 4d. cannot be taken; add 1s. or 12d., making 16d. in the minuend; 6d. from 16d. is 10d.

Then 19s. from 13s. cannot be taken; add £1 or 20s., making 33s. in the minuend; 19s. from 33s. is 14s.

Then £9 from £3 cannot be taken, but 9 from 13 is 4; and 3 from 6 is 3 for the tens' place; making £34.

£	s.	d.
63	13	4¼
28	18	5½
£34	14	10¾

Rule.—Write the subtrahend below the minuend so that each column shall be of the same name; subtract each column in its order, beginning with that of lowest name, and carrying as in compound addition; if any name in the minuend is less than the same name in the subtrahend, add to it one of the next highest name changed to its own, and add one to the next name in the subtrahend.

[1] Both methods of subtraction are given as in simple subtraction, sect. 25; the teacher may choose either.

EXERCISE I.

1. $7\frac{3}{4}-2\frac{1}{2}$, $5\frac{1}{2}-3\frac{1}{4}$, $8\frac{3}{4}-5\frac{3}{4}$, $10\frac{1}{2}-7$, $9\frac{1}{2}-1\frac{1}{2}$, etc.
2. $5\frac{1}{4}-3\frac{1}{2}$, $7\frac{1}{2}-6\frac{3}{4}$, $7\frac{1}{4}-5\frac{1}{4}$, $9\frac{1}{2}-7\frac{3}{4}$, $11\frac{1}{4}-8\frac{1}{2}$, etc.
3. $6/5-3/2$, $8/11-5/6$, $7/9-1/9$, $3/6\frac{1}{2}-2/4\frac{1}{2}$, $14/10\frac{1}{2}-7/4$, etc.
4. $4/3-2/6$, $7/2-3/8$, $8/4-4/7$, $8/4\frac{1}{4}-6/5$, $13/2\frac{3}{4}-8/8$, etc.

EXERCISE II.

1. Count back from 1/, 2/, etc., by $2\frac{1}{2}$d., $3\frac{1}{4}$d., $4\frac{3}{4}$d., etc.
2. Count back from 20/, 19/, etc., by 1/3, 1/4, 2/2, 1/7$\frac{1}{2}$, etc.
3. Count back from £5, etc., by 10/6, 12/8, 13/4$\frac{1}{2}$.

EXERCISE III.

61.

1.	2.	3.	4.	5.
£37 8 $4\frac{1}{2}$	£93 10 $3\frac{3}{4}$	£84 7 $10\frac{1}{4}$	£47 17 $8\frac{1}{2}$	£205 2 $9\frac{1}{2}$
19 5 $10\frac{3}{4}$	39 6 9	53 17 $1\frac{1}{4}$	29 8 $10\frac{3}{4}$	126 12 $8\frac{1}{4}$

6.	7.	8.	9.	10.
£730 2 $6\frac{1}{2}$	£704 14 $9\frac{1}{2}$	£294 0 $9\frac{1}{2}$	£360 10 $6\frac{3}{4}$	£545 12 $0\frac{1}{2}$
428 17 $8\frac{3}{4}$	396 0 $5\frac{1}{2}$	89 10 $9\frac{3}{4}$	219 19 $0\frac{3}{4}$	293 18 $0\frac{3}{4}$

11.	£848	0	$0\frac{3}{4}$	–	£274	10	1	19.	£8000	0	1	–	£1793	10	$0\frac{1}{4}$
12.	763	10	$11\frac{1}{2}$	–	294	18	$2\frac{3}{4}$	20.	3030	13	0	–	2594	0	$7\frac{1}{4}$
13.	540	0	$0\frac{1}{4}$	–	290	0	$0\frac{3}{4}$	21.	2000	0	$0\frac{1}{4}$	–	17	0	9
14.	643	15	$8\frac{1}{2}$	–	190	15	9	22.	903	0	$5\frac{1}{4}$	–	50	0	7
15.	1938	17	$6\frac{3}{4}$	–	209	19	$8\frac{1}{4}$	23.	1000	0	0	–	295	0	$7\frac{3}{4}$
16.	2467	14	$8\frac{1}{4}$	–	938	15	6	24.	3724	6	$10\frac{1}{4}$	–	1936	2	11
17.	3091	10	11	–	1857	16	$11\frac{1}{4}$	25.	5704	13	8	–	2945	2	$10\frac{3}{4}$
18.	4000	0	0	–	993	1	$1\frac{1}{4}$	26.	8407	0	$7\frac{1}{2}$	–	899	19	$9\frac{1}{4}$

EXERCISE IV.

Find the first remainder less than the subtrahend in—

1.	£2761	13	$4\frac{1}{2}$	–	£564	17	$6\frac{3}{4}$	10.	£8473	16	$4\frac{1}{4}$	–	£1005	5	$10\frac{1}{2}$
2.	4095	14	$0\frac{3}{4}$	–	709	19	1	11.	10000	0	0	–	2946	0	$5\frac{1}{4}$
3.	8740	0	$7\frac{1}{4}$	–	1096	10	$9\frac{1}{2}$	12.	7338	2	$11\frac{1}{2}$	–	943	4	$9\frac{1}{2}$
4.	5436	10	$8\frac{1}{4}$	–	854	12	$0\frac{3}{4}$	13.	7009	9	$0\frac{3}{4}$	–	856	0	$4\frac{3}{4}$
5.	9425	16	2	–	1906	17	$2\frac{1}{4}$	14.	1946	10	10	–	405	16	$11\frac{1}{4}$
6.	7464	13	11	–	948	17	$6\frac{3}{4}$	15.	6429	14	$6\frac{1}{2}$	–	842	15	$10\frac{1}{2}$
7.	4763	0	$0\frac{1}{4}$	–	742	11	4	16.	8754	12	$3\frac{3}{4}$	–	947	13	6
8.	6000	0	0	–	823	0	$0\frac{3}{4}$	17.	5431	18	$1\frac{1}{4}$	–	739	11	$4\frac{1}{4}$
9.	2346	2	10	–	473	0	$9\frac{1}{4}$	18.	9402	14	$8\frac{1}{4}$	–	1246	16	$8\frac{3}{4}$

62. COMPOUND MULTIPLICATION.

Ex.—What cost 9 chests of tea at £24, 14s. 7$\frac{1}{4}$d. per chest?

We have here to find 9 times the price of one chest; which we do by multiplication.

Write the multiplier under the pence column of the multiplicand.

Then, beginning with the lowest name, 9 times 1f. are 9f., which is 2¼d; set down 1f., and carry 2d.

£	s.	d.
24	14	7¼
		9
£222	11	5¼

9 times 7d. are 63, and 2 are 65d., which is 5/5; set down 5d., and carry 5/.

9 times 14s. are 126s., and 5 are 131s., which are £6, 11s.; set down 11s., and carry £6.

9 times 24 are 216, and 6 are £222.

Total product, £222, 11s. 5¼d.

The result may be proved by dividing the product by the multiplier (see sect. 65), which will give the multiplicand.

Rule.—When the multiplier is not above 12, multiply each name in the multiplicand by it in order, beginning with the lowest, and carry as in compound addition. When the multiplier is not greater than 144, and has two factors, neither above 12, multiply by each factor in succession.

The multiplication of quantities of different names, as here, is called *Compound Multiplication.*

EXERCISE I.

Multiply the following by 2, 3, 4, etc., up to 12 successively:—2¾d., 3¼d., 3¾d., 4½d., 5¼d., 5¾d., 6¼d., 6¾d., 7¼d., 7½d., 8¼d., 8¾d., 9¼d., 9½d., 10½d., 10¾d., 11¼d., 11½d., 11¾d.

EXERCISE II.

Multiply by 2, 3, 4, etc., to 12 successively:—

1. 6d., 8d., 10d., 1/1, 1/4, 1/8, 2/1, 2/7, 3/4, 3/6, 4/2, 4/10, 5/6, etc.
2. 10/1, 10/3, 10/9, 11/, 12/2, 12/8, 13/3, 13/10, 14/4, 15/1, 15/11, etc.

EXERCISE III.

1. 2/4½, 5/8¼, 7/0¾, 8/11¼, 11/4¾, 14/6½, 16/9¼, 17/2½, . . × 2.
2. 3/2¾, 4/2¾, 5/6½, 6/7½, 7/8¼, 13/4¾, 18/2½, 19/1¾, . . × 3.
3. 2/9¼, 3/2¾, 6/3¼, 8/2¾, 10/7½, 12/4¾, 19/1¾, 14/6½, . . × 4.
4. 1/8¾, 2/11¾, 5/11½, 9/3½, 16/4¼, 13/2¼, 11/6½, 13/7¼, . . × 5.
5. 2/2¼, 4/7½, 7/2¼, 8/7¾, 14/4¾, 16/3¼, 17/9¼, 18/11, . . × 6.
6. 3/7, 4/2¼, 1/10½, 5/6¼, 8/7¾, 9/10½, 15/8¾, 16/9½, . . × 7.
7. 4/5½, 5/11, 6/11¼, 16/4½, 9/2¾, 12/8½, 9/7½, 13/2¾, . . × 8.
8. 3/0½, 3/10½, 18/4½, 5/11½, 12/6¾, 9/7½, 14/10½, 17/0¾, . × 9.
9. 1/5¾, 2/7¾, 6/3½, 9/8½, 11/7¾, 15/8½, 10/5¼, 12/8½, . . ×11.
10. 3/1¼, 5/11½, 1/6½, 3/5¾, 8/2¼, 13/9½, 16/8½, 19/3½, . . ×12.

EXERCISE IV.

1. £7, 8, 4½×2, 4, 7, 8, 9.
2. £10, 9, 4×3, 6, 8, 10, 11.
3. £16, 0 5¾×4, 5, 7, 9, 12.
4. £21, 4. 8¼×3, 8, 2, 7, 5.
5. £34, 17, 11½×7, 11, 9, 12, 3.
6. £43, 10, 10¼×5, 8, 4, 10, 7.

7. £87, 9, $0\frac{3}{4}$ ×14, 15, 21, 22.
8. £92, 19, $4\frac{1}{2}$ ×25, 27, 28, 32.
9. £127, 5, $6\frac{1}{7}$ ×35, 42, 44.
10. £209, 15, $7\frac{3}{4}$ ×45, 48, 54.
11. £543, 18, $2\frac{1}{4}$ ×56, 60, 63.
12. £708, 13, $1\frac{3}{4}$ ×64, 72, 77.
13. £900, 0, $9\frac{1}{7}$ ×84, 99, 108.
14. £1256, 10, $0\frac{3}{4}$ ×121, 132, 144.

*** Multiply by three factors.

15. £18, 9, $4\frac{1}{4}$ ×112, 125.
16. £37, 0, $9\frac{3}{4}$ ×105, 126.
17. £85, 17, $2\frac{1}{2}$ ×192, 216.
18. £90, 14, $8\frac{3}{8}$ ×128, 135.
19. £74, 8, $11\frac{1}{2}$ ×147, 162.
20. £105, 15, $0\frac{1}{4}$ ×189, 210

63. Multipliers of Two Places.

Ex.—Find the price of 68 chests tea at £24, 14s. $7\frac{1}{4}$d. per chest.

The number 68 cannot be resolved into two factors under 12. Take the next less which can, that is, 64. Find the price of 64 chests (8×8), and add the price of 4 chests; for 64=8×8+4.

£	s.	d.	
24	14	$7\frac{1}{4}$×4	
		8	
197	16	10	price of 8 chests.
		8	
1582	14	8	„ 64 chests.
98	18	5	„ 4 chests.
1681	13	1	„ 68 chests.

The price of 64 chests is found as above: the price of 4, by multiplying the price of one (first line) by 4; the price of 68 by adding the price of 64 and the price of 4 together.

Other factors which might be used are $9 \times 7 + 5$ and $10 \times 6 + 8$, either of which pairs may be taken to prove the result.

Rule.—When the multiplier is not above 144, and cannot be resolved into two factors under 12, multiply by the factors of the next less number which has them, and add the product of the multiplicand by the difference between that number and the multiplier.

It is advisable to take factors for the number next *above* the multiplier, when that number exceeds it only by 1, and then subtract the excess; thus, 39=10×4−1. In the present case we might have taken 68=10×7−2.

EXERCISE V.

1. £2, 14, $2\frac{3}{4}$ × 13, 17, 19, 24, 29, 31.
2. £7, 10, $9\frac{1}{4}$ × 34, 38, 43, 51, 58, 61.
3. £13, 8, $5\frac{1}{2}$ × 62, 69, 74, 78, 82, 87.
4. £34, 3, $2\frac{1}{4}$ × 91, 94, 101, 106, 117, 122.
5. £60, 0, $9\frac{3}{4}$ × 129, 135, 142, 145.
6. £79, 18, $6\frac{1}{2}$ × 67, 79, 46, 39, 89, 105.

64. Multipliers of Three Places.

Ex.—Find the price of 457 chests at £24, 14s. 7¼d. per chest.

£24	14	7¼ × 7 =	£173	12	2¾	price of 7 chests.				
		10								
£247	6	0½ × 5 =	1236	10	2½	„	50	„		
		10								
£2475	0	5 × 4 =	9900	1	8	„	400	„		
		Total product,	£11310	4	2¼	„	457	„		

Rule.—Multiply by factors for 100 (10×10). Then multiply the *multiplicand* by the number of *units* in the multiplier, *ten times the multiplicand* by the number of *tens* in it, and a *hundred times the multiplier* by the number of *hundreds* in it: add these three products for the total product.

EXERCISE VI.

1. £9, 13, 7½ × 257, 381, 473.
2. £13, 10, 8¼ × 319, 459, 542.
3. £19, 8, 5¼ × 417, 534, 629.
4. £23, 10, 0¾ × 566, 671, 713.
5. £31, 19, 4¼ × 647, 738, 825.
6. £43, 1, 11¼ × 724, 850, 993.
7. £59, 7, 3¼ × 915, 638, 187.
8. £73, 8, 10½ × 562, 784, 268.
9. £83, 15, 7¼ × 400, 701, 511.
10. £89, 0, 5½ × 208, 962, 609.
11. £93, 14, 2¾ × 354, 849, 276.
12. £109, 7, 9¼ × 417, 651, 767.

Multipliers of Four Places.

The same method is used for multiplying by thousands.

Rule.—Multiply by factors for 1000 (10×10×10). Then multiply the multiplicand and the successive products by the several places of the multiplier in order, beginning with the units' place; add these products for the total product.

EXERCISE VII.

1. £13, 18, 5⅓ × 1924, 2438.
2. £19, 5, 10¼ × 2741, 3925.
3. £27, 3, 4¾ × 4837, 5529.
4. £57, 10, 7¾ × 6234, 7941.
5. £69, 5, 8½ × 8301, 9042.
6. £124, 15, 6¼ × 4520, 6009.

*** These products are obtained more easily by *practice*.

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65. COMPOUND DIVISION.

Ex. 1.—Divide £93, 15s. 9¾d. equally among 7 persons: what is the share of each?

Write the divisor and dividend as in simple division.

Then 7 in £93 is £13 and £2 over; set down the £13, and carry the £2 to the shillings, making 55s. in all.

$$7\overline{)93\ 15\ 9\tfrac{3}{4}}$$
$$13\ \ 7\ \ 11\tfrac{1}{2}\tfrac{5}{7}$$

7 in 55 is 7s. and 6s. over; set down the 7s. and carry the 6s. to the pence, making 81d. in all.

7 in 81 is 11d. and 4d. over; set down the 11d. and carry the 4d. to the farthings, making 19 farthings in all.

7 in 19 is 2 farth. and 5 farth. over; set down the 2 farth. and, as the division is now finished, there is a remainder of 5 farthings, divided thus, $\frac{5}{7}$.

Quotient, £13, 7s. $11\frac{1}{2}\frac{5}{7}$.

The result may be proved by multiplying the quotient by the divisor, and adding the remainder, which will give the dividend.

Ex. 2.—Divide the same sum equally among 28 persons.

Resolve the divisor into its two factors (7×4), and divide by each in succession.

$$\begin{array}{r|l} 7 & 93\ 15\ 9\frac{3}{4} \\ 4 & 13\ \ 7\ \ 11\frac{1}{2}+5 \\ & 3\ \ 6\ \ 11\frac{3}{4}+2 \end{array}\Big\}\ 19\text{f.}$$

Quotient, £3, 6s. $11\frac{3}{4}\frac{19}{28}$.

The result may be proved by reversing the order of factors in dividing, or by multiplying the product by the divisor.

Rule.—When the divisor is not above 12, divide each name by it in order, beginning at the highest, and carry the remainder to the next lower. When the divisor is not above 144, and has two factors neither above 12, divide in the same way by each factor in succession.

The division of a quantity of several names, as here, is called *compound division*.

66. EXERCISE I.

1. 2d. 3d. 5d. 6d. 7d. etc.÷2, 4.
2. 1½d. 3d. 4½d. 6d. 7½d. etc.÷3, 6.
3. 1¼d. 2½d. 3¾d. 5d. 6¼d. etc.÷5.
4. 1¾d. 3½d. 5¼d. 7d. etc.÷7.
5. 2d. 4d. 6d. 8d. etc.÷8.
6. 2¼d. 4½d. 6¾d. 9d. etc.÷9d.
7. 1/, 1/2, 1/4, 1/8, 1/10, 2/, etc.÷2, 4.
8. $1\frac{1}{4}$, $1/1\frac{1}{2}$, $1/3\frac{3}{4}$, 1/6, $1/8\frac{1}{4}$, etc.÷3, 9.
9. $1/0\frac{1}{2}$, 1/3, $1/5\frac{1}{2}$, 1/8, $1/10\frac{1}{2}$, etc.÷5.
10. 1/3, 1/6, 1/9, 2/, 2/3, etc.÷6, 12.
11. $1/0\frac{1}{4}$, 1/2, $1/3\frac{3}{4}$, $1/5\frac{1}{2}$, etc.÷7.
12. $1/1\frac{1}{8}$, $1/4\frac{1}{2}$, $1/7\frac{1}{4}$, 1/10, etc.÷11.
13. £1, £1, 4, £1, 8, etc.÷2, 4, 8.
14. £1, 2/6, £1, 5/6, £1, 8/6, etc.÷3, 9.
15. £1, £1, 5, £1, 10, etc.÷5, 10.
16. £1, 4, £1, 10, £1, 16, etc.÷6, 12.
17. £1, 1, £1, 4/6, £1, 8, etc.÷7.
18. £1, 2, £1, 7/6, £1, 13, etc.÷11.

EXERCISE II.

1.	£8	19	$7\frac{3}{4}$	÷2, 3, 4, 5.	13.	£89	14	$10\frac{3}{4}$	÷14, 15, 21.
2.	7	0	$5\frac{1}{2}$	÷3, 4, 5, 6.	14.	91	2	$8\frac{1}{4}$	÷24, 27, 22.
3.	19	10	$3\frac{1}{4}$	÷4, 5, 6, 7.	15.	156	17	$3\frac{1}{4}$	÷25, 28, 100.
4.	27	15	$6\frac{1}{2}$	÷5, 6, 7, 8.	16.	193	0	5	÷30, 32, 108.
5.	79	1	$11\frac{5}{6}$	÷6, 7, 8, 9.	17.	279	6	$10\frac{1}{2}$	÷84, 96, 99.
6.	54	0	$0\frac{1}{4}$	÷7, 8, 9, 10.	18.	309	1	$4\frac{3}{5}$	÷80, 81, 35.
7.	60	5	$7\frac{1}{2}$	÷8, 9, 10, 11.	19.	600	10	$10\frac{1}{4}$	÷77, 72, 121.
8.	86	14	$9\frac{3}{4}$	÷9, 10, 11, 12.	20.	793	15	$6\frac{3}{5}$	÷70, 64, 18.
9.	43	6	$11\frac{1}{4}$	÷10, 11, 12, 7.	21.	72	5	$6\frac{1}{4}$	÷56, 63, 16.
10.	37	18	$1\frac{3}{5}$	÷11, 12, 5, 9.	22.	68	7	$3\frac{1}{2}$	÷48, 50, 144.
11.	5	17	5	÷12, 6, 7, 10.	23.	81	19	$0\frac{3}{4}$	÷42, 44, 132.
12.	3	12	$9\frac{1}{4}$	÷7, 9, 4, 5.	24.	59	2	$7\frac{1}{2}$	÷36, 40, 33.

67. Divisors of Two or more Places.

Ex.—Divide £93, 15s. $9\frac{3}{4}$d. among 43 persons.

Rule.—Divide each name in order by the divisor, beginning at the highest; and carry each remainder to the next lower name.

```
43)93  15   9¾ (2   3   7¼  40/43
   86
   ---
    7
   20
   ---
  )155 s.
  129
  ---
   26
   12
   ---
  )321 d.
  301
  ---
   20
    4
   ---
  )83 f.
   43
   ---
   40
```

*** The 40 farthings over are written in the quotient with the divisor below them, as $\frac{40}{43}$.

EXERCISE III.

1.	£567	10	$3\frac{1}{2}$	÷29, 37, 53, 71, 83.
2.	734	18	5	÷19, 41, 67, 86, 91.
3.	392	15	$4\frac{1}{2}$	÷52, 23, 47, 95, 13.
4.	78	2	$11\frac{1}{4}$	÷124, 213, 352, 793, 61.
5.	27	18	$0\frac{1}{2}$	÷225, 538, 401, 191, 17.
6.	115	0	$10\frac{3}{4}$	÷115, 116, 237, 73, 85.
7.	1897	14	$3\frac{1}{4}$	÷372, 416, 509, 1000, 1937.
8.	2700	18	$0\frac{3}{4}$	÷562, 57, 829, 900, 2340.
9.	8035	17	$5\frac{1}{2}$	÷1256, 4073, 236, 800, 158.
10.	5682	11	$3\frac{1}{4}$	÷721, 1356, 2943, 673, 78.
11.	73582	14	$7\frac{3}{4}$	÷2905, 7238, 825, 34, 304.
12.	290732	9	$1\frac{1}{2}$	÷59, 97, 652, 8905, 4005.

68. Fractional Multipliers.

Ex.—What cost $8\frac{3}{4}$ packages if 1 package cost £5, 17s. $9\frac{1}{4}$d.?

Multiply first by the fraction ($\frac{3}{4}$), then by the whole number (8). Add the products.

£	s.	d.	
5	17	$9\frac{1}{4}$	
		$8\frac{3}{4}$	
4)17	13	$3\frac{3}{4}$	
4	8	$3\frac{3}{4}$	$\frac{3}{4}$
47	2	2	
£51	10	$5\frac{3}{4}$	$\frac{3}{4}$

EXERCISE IV.

1. £7, 10, $3\frac{1}{2} \times 7\frac{1}{4}$, $9\frac{1}{2}$, $11\frac{3}{4}$.
2. £14, 15, $7\frac{3}{4} \times 4\frac{3}{4}$, $6\frac{3}{8}$, $8\frac{1}{2}$.
3. £24, 19, $3 \times 15\frac{1}{2}$, $27\frac{3}{4}$, $36\frac{3}{4}$.
4. £71, 5, $11\frac{1}{2} \times 49\frac{3}{4}$, $84\frac{1}{2}$, $100\frac{1}{2}$.
5. £91, 15, $6\frac{1}{2} \times 73\frac{1}{4}$, $59\frac{1}{4}$, $91\frac{3}{4}$.
6. £256, 14, $10 \times 29\frac{3}{4}$, $13\frac{1}{2}$, $63\frac{1}{4}$.
7. £509, 8, $3\frac{3}{4} \times 231\frac{1}{2}$, $450\frac{1}{4}$, $671\frac{3}{4}$.
8. £891, 11, $1\frac{1}{4} \times 307\frac{3}{8}$, $593\frac{1}{2}$, $713\frac{3}{4}$.

Fractional Divisors.

Ex.—If $17\frac{3}{4}$ yards cost £9, 18s. $10\frac{1}{2}$d., what is that per yard?

We have to divide the whole price by the number of yards to get the price of one yard.

Multiply both divisor and dividend by 4 to remove the fraction from the divisor.

$17\frac{3}{4}$ × 4 = 71

£9 18 $10\frac{1}{2}$ × 4

)39 15 6 (11s. $2\frac{1}{4}\frac{57}{71}$

20
795
71
85
71
14
12
174
142
32
4
)128
71
57f.

EXERCISE V.

1. £7, 10, $11\frac{3}{4} \div 5\frac{1}{2}$, $6\frac{3}{8}$, $9\frac{3}{8}$.
2. £11, 14, $5\frac{1}{4} \div 8\frac{1}{4}$, $15\frac{3}{4}$, $49\frac{1}{2}$.
3. £29, 5, $0\frac{1}{4} \div 18\frac{3}{4}$, $29\frac{1}{4}$, $63\frac{1}{4}$.
4. £36, 7, $2\frac{3}{4} \div 21\frac{1}{4}$, $87\frac{1}{2}$, $52\frac{3}{4}$.
5. £58, 16, $1\frac{1}{2} \div 130\frac{1}{4}$, $200\frac{1}{2}$, $563\frac{1}{4}$.
6. £251, 17, $4\frac{3}{4} \div 117\frac{3}{4}$, $352\frac{1}{2}$, $401\frac{3}{8}$.
7. £309, 19, $2 \div 308\frac{3}{8}$, $510\frac{1}{4}$, $713\frac{1}{4}$.
8. £643, 0, $5\frac{1}{4} \div 83\frac{1}{4}$, $173\frac{1}{2}$, $824\frac{3}{4}$.

69. Money Divisors.

Ex.—How often is £5, 13s. $6\frac{1}{4}$d. contained in £39, 14s. $7\frac{3}{4}$d.?

Rule.—Reduce divisor and dividend to the same name, and proceed as in simple division—

£39, 14, $7\frac{3}{4}$ ÷ £5, 13, $6\frac{1}{4}$ = 38143f. ÷ 5449 = 7.

EXERCISE VI.

*** To be performed after reduction has been learnt.

1. £27, 17, 3 ÷ £6, 3, 10.
2. £137, 8, 9 ÷ £8, 19, $4\frac{1}{2}$.
3. £361, 2, $9\frac{3}{4}$ ÷ £72, 4, $6\frac{3}{4}$.
4. £2090, 0, $7\frac{1}{2}$ ÷ £81, 0, $9\frac{1}{4}$.
5. £459, 18, 2 ÷ £24, 17, $8\frac{1}{2}$.
6. £63, 8, $0\frac{3}{4}$ ÷ £21, 2, $8\frac{1}{4}$.
7. £671, 10, 1 ÷ £47, 19, $3\frac{1}{2}$.
8. £268, 10, 3 ÷ £100, 9, $10\frac{3}{4}$.
9. £675, 19, $3\frac{3}{4}$ ÷ £75, 2, $1\frac{1}{4}$.
10. £870, 0, $5\frac{3}{4}$ ÷ £39, 18, $5\frac{1}{2}$.

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70. REDUCTION.

MONEY OF ACCOUNT—TABLE I.

From a Higher to a Lower Name.

Ex.—In £7, 13s. $3\frac{3}{4}$d., how many farthings?

We cannot change this sum to farthings by one step, as it is too large; we must therefore do it in parts, changing first the pounds to shillings, then the shillings to pence, then the pence to farthings.

Thus, to change the pounds to shillings, since there are 20/ for every pound, there will be 20 times as many shillings as pounds; multiply 7 by 20, and add the 13/ already in the sum, making 153 sh.

£	s.	d.	
	7	13	$3\frac{3}{4}$
	20		
	153	$3\frac{3}{4}$	sh. in the sum.
	12		
	1839	$\frac{3}{4}$	pence in the sum.
	4		
	7359		farth. in the sum.

To change the shillings to pence, since there are 12d. in every shilling, there will be 12 times as many pence as shillings; multiply 153 by 12, and add the 3d. already in the sum, making 1839d.

To change the pence to farthings, since there are 4 farth. in every penny, there will be 4 times as many farthings as pence; multiply 1839 by 4, and add the 3 farth. already in the sum, making 7359f. in all.

Rule.—Multiply each name, in order from the highest, by the number of the next lower which it contains, adding to each product the number of the lower in the given sum.

The process of changing from one name to another is called *Reduction*.

The result may be proved by changing back the farthings to pounds; dividing by the same numbers by which we have multiplied. If £7, 13s. $3\frac{3}{4}$d., when changed to farthings gives 7359f., 7359 farthings, when changed to pounds, must give £7, 13s. $3\frac{3}{4}$d. (See sect. 71.)

EXERCISE I.

1. How many farthings in $1\frac{1}{4}$d., $1\frac{1}{2}$d., $1\frac{3}{4}$d., 2d., $2\frac{1}{4}$d., etc., to 12d.?
2. How many pence in 1/1, 1/2, etc., 2/1, 2/2, etc., to 20?
3. How many shillings in £1, 1s.; £1, 5s., etc.; £2; £2, 7s; £10?

EXERCISE II.

(1.) To pence—£75; £352; £1001; £2450; £23, 10s; £179, 17s.; £305, 19s.; £5024, 15s.; £734, 17s. 4d.; £809, 10s. 8d.; £2702, 0s. 11d.; £6304, 1s. 7d.

(2.) To halfpence—5/, 7/, 13/, 8/2, 18/3, 14/7$\frac{1}{2}$, 53/8$\frac{1}{2}$, £15, £23, 17s., £27, 9s. 10d., £150, 0s. 7$\frac{1}{2}$d., £207, 19s. 0$\frac{1}{2}$d.

(3.) To farthings—4/, 9/, 24/, 37/, 3/4$\frac{1}{2}$, 11/9$\frac{1}{4}$, 19/1$\frac{1}{4}$, 15/0$\frac{3}{4}$, 29/10$\frac{3}{4}$, 72/8$\frac{3}{4}$, 13/9$\frac{1}{4}$, 194/0$\frac{1}{2}$.

(4.) To farthings—

1.	£93.	5.	£39, 17.	9.	£4, 17, 10.	13.	£922, 10, 0$\frac{1}{2}$.
2.	£201.	6.	£125, 8.	10.	£172, 0, 0$\frac{1}{4}$.	14.	£507, 19, 11$\frac{3}{4}$.
3.	£485.	7.	£709, 10.	11.	£250, 0, 0$\frac{3}{4}$.	15.	£1854, 0, 3.
4.	£7392.	8.	£4890, 19.	12.	£793, 15, 11$\frac{1}{4}$.	16.	£3000, 10, 10$\frac{1}{4}$.

71. From a Lower to a Higher Name.

Ex.—To what sum of money are 37227 farthings equivalent?

Here we have to change the farthings to the highest name.

We cannot do this at one step, as the number is too large; we must therefore do it by several steps, first changing the farthings to pence, then the pence to shillings, then the shillings to pounds, thus:—

To change for the farthings to pence, since it takes 4 farthings to make 1 penny, there be only one-fourth as many pence as farthings; which is got by dividing the number of pence by 4, giving 9306$\frac{3}{4}$d.

4	37227	
12	9306$\frac{3}{4}$	= pence in the sum.
2(0	77(5 $6\frac{3}{4}$	= shillings, etc. in sum.
	£37 15 $6\frac{3}{4}$	= pounds, etc. in sum.

To change the pence to shillings, since it takes 12 pence to make 1 shilling, there will be only one-twelfth as many shillings as pence; which is got by dividing by 12, giving 775s. 6$\frac{3}{4}$d.

To change the shillings to pounds, since it takes 20 shillings to make 1 pound, there will be only one-twentieth as many shillings as pounds; which is got by dividing by 20, giving £37, 15s. 6¾d.

Rule.—To change a sum of money from a lower to a higher name:—Divide by the number of the lower contained in the next higher, and so on till the required name be reached.

The result may be proved by changing back the pounds, shillings, and pence to farthings. If 37227f., when changed, give £37, 15s. 6¾d., so must £37, 15s. 6¾d., when changed back again to farthings, give 37227f.

EXERCISE III.

1. How many pence in 4 f. 5, 6, 7, etc., to 48 f.
2. How many shillings in 12d., 13d., etc., 24d., 25d., etc., to 240d.
3. How many £ in 20/, 40/, etc., 21/, 22/, etc., 30/, 31/, etc., to 200/.

EXERCISE IV.

1. To shillings from farthings—912, 1344, 1680, 2352, 737, 501, 1079, 1893, 600, 903, 1807, 2356.
2. To shillings from halfpence—360, 432, 552, 768, 247, 301, 423, 593, 827, 1327, 1613, 2597.
3. To pounds from pence—6480, 2376, 4800, 11040, 35721, 60089, 23459, 45930, 49087, 780923, 56421, 93000.
4. To pounds from farthings—23496, 39408, 45082, 69857, 289508, 543306, 60085, 932092, 1000000, 2456793, 4560000, 5369480.